Part III – Minor Option in Medical Physics 2018
Examples Sheet

Any errors or comments should be addressed sent to: seb53@cam.ac.uk

URLs that may be useful:
Stanford Event Generation Simulator: http://tinyurl.com/pkg476r
NIST:
Tabulation of Attenuation Coefficients: http://tinyurl.com/ofowysd
Tabulation of Relevant Biological Attenuation Coefficients: http://tinyurl.com/prvp3fb

1. Radiation Interactions and Image Quality Fundamentals (Lectures 1-2)

1.1

a) Briefly describe the processes by which an X-ray photon might interact in tissue at diagnostic energies.
b) How does the probability of these processes depend on the photon energy and the type of tissue in question?
c) Applying conservation of energy and momentum for the relativistic Compton scattering, derive the relationship for E_1/E_0 given in lectures and plot a graph of the scattered angle as a function of energy.
d) How do Compton scattered photons contribute to X-ray image noise?

1.2

Use the Stanford Event Generation Simulator (http://tinyurl.com/pkg476r) to help you to compare and contrast how charged particles and photons interact in the body. You may assume that the body is composed entirely of water. How deep does an electron beam of energy 100keV, 1MeV or 10MeV penetrate compared to a photon beam of the same energy? How many photons per electron will one observe behind the body at each of these energies, assuming a body thickness of 30cm?

Hints: You will need to use the zoom and move options on the generated plots to see the tracks of low energy electrons. It may help to generate electrons without allowing a vertical spread in the y axis.

1.3

Image quality metrics are best understood using an illustrative worked example. Please see the end of this document for the full instructions on how to complete this question.

2. Imaging with non-ionising radiation (Lectures 3-7)

Ultrasound

2.1

Using the data in the table (next page), calculate the intensity transmission coefficient for the following interfaces, assuming i. normal incidence and ii. 45 degree incidence of the ultrasound beam:

a) Muscle/kidney.
b) Air/muscle.
c) Bone/muscle.

Discuss the implications of these values for ultrasound imaging.

Tissue	Z x 10^5 ($gcm^{-2}s^{-1}$)	c (ms^{-1})	Density (kgm^{-3})	Compressibility x 10^{11} ($cmg^{-1}s^2$)
Muscle	1.7	1590	1075	3.7
Kidney	1.62	1560	1040	4.0
Air	0.00043	330	1.3	70000
Bone	7.8	4000	1908	0.3

2.2
To improve the efficiency of ultrasound transducers, we attempt to minimise the amount of energy reflected between the PZT crystal and the object of interest using impedance matching.

a) For a layer of acoustic impedance Z_M placed between the transducer (Z_{PZT}) and skin (Z_{skin}), show that the value of Z_M that minimises the reflected energy is

$$Z_M = \sqrt{Z_{PZT} Z_{skin}}$$

b) Given values of Z_{PZT}=30x10^5 $gcm^{-2}s^{-1}$ and Z_{skin}=1.7x10^5 $gcm^{-2}s^{-1}$, calculate what fraction of transducer energy is transmitted.

c) If a transducer element has a thickness of one-half of the wavelength of ultrasound in the crystal (to produce resonance), show that the thickness of the matching layer between the transducer and the body should be one-quarter of the ultrasound wavelength.

2.3
Show that, in the scan plane, the beam of a single 1mm wide element of a 3MHz linear array has a near field length of only about 0.5mm and then diverges between angles ± 30 degrees to the axis. How would the beam be modified if it were produced by an active group of 20 adjacent elements?

2.4

a) Explain the process of image formation in B-mode and Doppler.
b) For each of these imaging modes, give two examples of common artifacts that are observed in the images.
c) How can these artifacts be avoided or minimised?
d) Which of these imaging modes has the greatest potential to do harm to the patient and why?
e) How can the ultrasound operator minimise the risk of harm to the patient?

MRI

2.5

a) Assuming that there are 6.7 x 10^{22} protons in a cubic centimetre of water, what is the magnetization contained within this volume at 3T?
Note: $M_0 = \mu_z(N_{parallel}-N_{anti\text{-}parallel})$

b) Explain briefly why protons are particularly suitable for nuclear magnetic
resonance imaging (MRI). Where are protons found in the body?

c) What is a free induction decay? Why does it persist for a long time in a homogeneous B_0 field but decay more rapidly in an inhomogeneous field?

d) A tissue with T1=500 ms and T2=50 ms is initially at equilibrium when the following sequence of RF pulses is applied to it: A 180 degree pulse, a gap of 250 ms, a 90 degree pulse, a gap of 500 ms and a final 90 degree pulse. What would be the relative size of the free induction signals following each 90 degree pulse?

2.6

a) Draw and annotate a gradient echo pulse sequence diagram with the following: TR, TE, RF pulse, Gslice, Gfrequency, Gphase and explain how an image is formed as a result.

b) Why is it necessary to have an opposite polarity gradient pulse immediately after the slice selection gradient? Approximately what area should this pulse have in comparison to the slice select gradient?

c) Water has a T_1 of 2.5s and a T_2 of 250ms. Give appropriate estimates of TR, TE and flip angle in the gradient echo sequence to produce:
   i. A T_1 weighted image
   ii. A T_2^* weighted image
   iii. A proton density weighted image

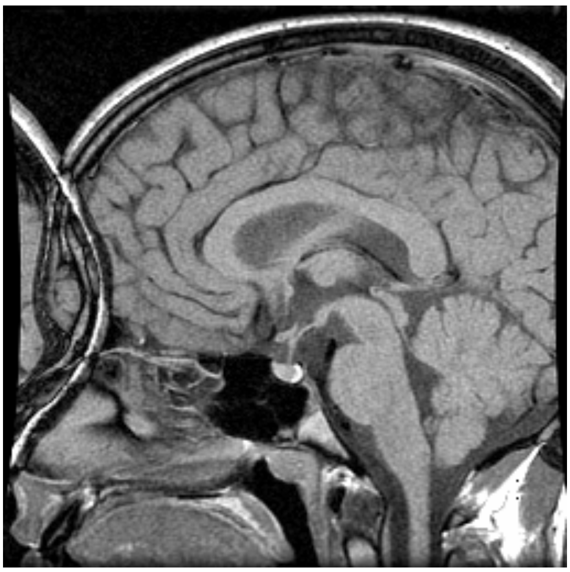
d) For each image type in part (b), explain which type of tissue would have the highest signal intensity.

e) How would your answer to (b) change if this were a spin echo sequence?

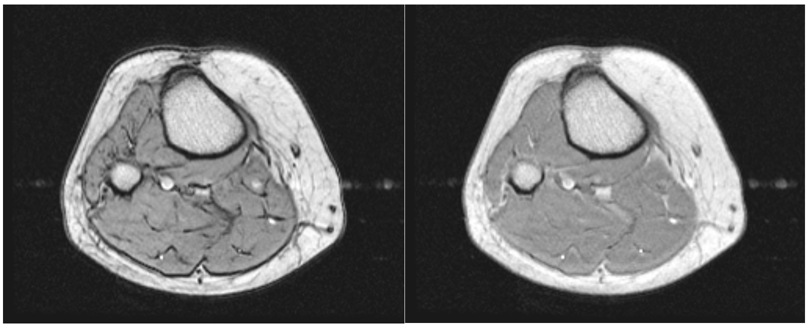
f) Will T_1 and T_2 increase or decrease if the temperature is increased? Justify your answer.

2.7

The figure below shows a sagittal spin echo image of the brain. Explain why the back of the subjects' head appears on the left-hand side of this image.



2.8
Due to magnetic shielding of the surrounding electron cloud, the hydrogen nuclei in fat precess approximately 3.4ppm slower than the hydrogen nuclei in water. In a gradient echo sequence this will result in fat and water becoming periodically 180° out-of-phase and then in-phase. What are the minimum TEs at 1.5T where fat and water are out-of-phase and then in phase. *Hint: The Larmor frequency at 1.5T is approximately 64MHz.*



Out-of-phase In-phase

<u>3. Imaging with ionising radiation (Lectures 8-12)</u>

3.1
What is meant by the term effective dose? What levels of effective dose might typically received by patients undergoing a radiographic x-ray examinations of the head compared to a CT of the head? How might you justify such an exposure to the concerned parents of a child about to have an x-ray?

3.2

a) Outline the difference between stochastic and deterministic radiation effects.
b) Describe how radiation protection can be applied to limit these effects.

3.3
A bremsstrahlung beam of polyenergetic photons with maximum energy E_{max} = 100 keV has an energy distribution given by:

$$\frac{dN(E)}{dE} = C\left(1 - \frac{E}{E_{\max}}\right)$$

for $E<E_{max}$ where $C=10keV^{-1}$. The beam propagates through an aluminium plate for thickness 0.5 cm.

a) Make a plot of the initial and final (after plate) distributions of the photons dN/dE as a function of E using the NIST tables for attenuation coefficients (see links to NIST tabulations at start of document). *Note: this is most simply approached from a spreadsheet of dN/dE values.*
b) Write an expression for the average photon energy and estimate the value before and after the plate, as well as the fraction of the photons that interacted in the plate
c) Repeat (a) and (b) for
    a. Different thicknesses of aluminium
    b. Lead and Beryllium

d) Comment on the use of Al, Be and Pb in diagnostic X-ray imaging in relation to properties you have derived.

3.4

A significant amount of natural radiation comes from the decay of Radium-226. The decay chain involves a sequence of alpha and beta decays. The final steps in the decay chain are:

$$^{210}Bi \rightarrow {}^{210}Po \rightarrow {}^{206}Pb$$

The first step is a beta decay with half-life of 5 days, while the second is an alpha decay with a half-life of 138 days.

a) If at t=0 we have no ^{210}Bi or ^{206}Pb, but 1 mole of undecayed ^{210}Po, what is the activity of the source at t=0 and t=84 days?
b) If at t=0 there are N_0 nuclei of ^{210}Bi and zero nuclei of the other types, find expressions for the number of nuclei of each kind as a function of time.
c) Hence find the time at which the ^{210}Po population is maximum.

3.5

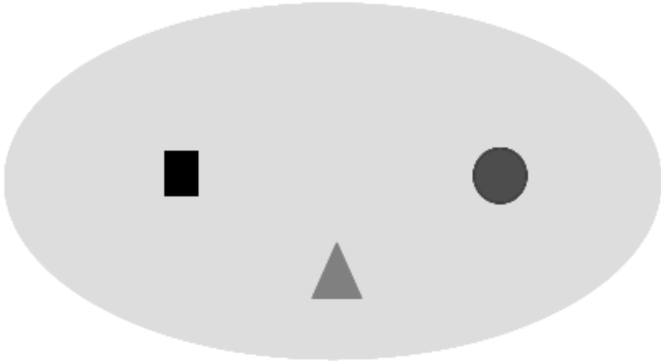
a) Explain how a scintillation detector works and how it can be used to measure both the intensity and the energy of a beam of radiation.
b) Discuss the factors that have to be considered when selecting a scintillator for a particular imaging application.
c) Describe the sequence of events that leads to a pulse of electrons at the anode of a photomultiplier tube if the tube is directed at a NaI scintillation crystal placed in a beam of gamma rays.

3.6

a) Outline the difference between direct and indirect methods of X-ray detection and explain how these are used in analogue (film) and digital X-ray imaging.
b) What is meant by the depletion layer in a p-n diode? How does it arise and how can it be used to detect light?
c) What are advantages of digital X-ray imaging in comparison to film? Your answer should include comments relating to the standard image quality metrics, such as SNR and MTF.

3.7

For the object shown below, draw the projections at angles of 0, 45, 90 and 135 degrees then draw the sinogram produced by the object from 0 to 360 degrees. Assume that a dark area in the object corresponds to an area of high signal.



3.8

a) Write brief notes on back projection methods for image reconstruction of tomographic data
b) Backproject the following, commenting on the image produced and suggest ways it could be improved.

0	0	0	0	0
0	0	0	0	0
0	0	3	0	0
0	0	0	2	0
0	0	0	0	0

3.9

Describe the differences between rigid (including affine) and non-rigid registration techniques. Give an example case where each technique would be used

3.10

a) Using the rest mass of the electron, show that the energies of the two gamma rays produced by annihilation of an electron with a positron are 511 keV.
b) Explain the general principles of positron emission tomography (PET), referring to the source of contrast, the data collection procedure and the process of image formation in your answer.
c) Making reference to attenuation corrections, explain why PET images often show an artificially high level of radioactivity in the lungs.

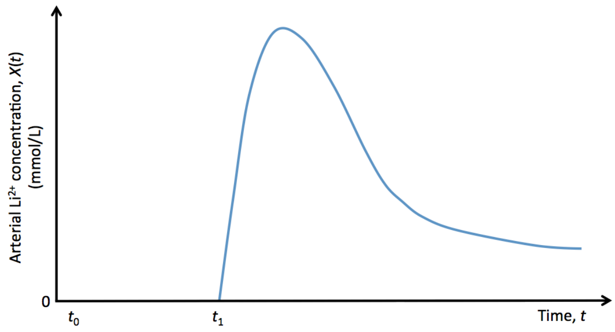
4. Physics beyond Medical Imaging (Lectures 14-16)

4.1

Blood pressure is often used clinically to assess the function of the cardiovascular system.

a) Why is the delivery of oxygen to tissues important? What factors determine oxygen delivery? Why is blood pressure a poor surrogate for oxygen delivery?

b) A known quantity Q_{Li} (mmol) of lithium ions is injected into the venous system at time t_0. The figure below shows the concentration $X(t)$ of lithium in arterial blood as measured with a lithium sensitive electrode. What is the significance of the time interval t_1-t_0?



c) Assuming that lithium is a predominantly extracellular ion, show that the cardiac output C is given by

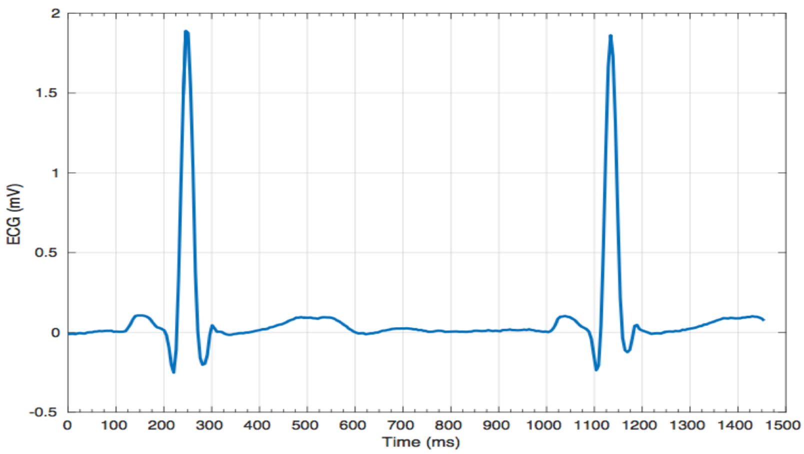
$$C = \frac{60Q_{Li}}{\left(1 - f_{PCV}\right)\int_{t_1}^{\infty} X(t)\,dt}$$

where f_{PCV} is the packed cell fraction of whole blood.

d) The *X(t)* curve shown does not appear to return to zero. Why might this be? Is this a problem?

4.2

The figure below shows a short segment of ECG (lead II) recording from a real patient.



a) Briefly describe from where the ECG signal originates within the body and how it is measured. How is it used clinically? What does the term “lead” refer to?

b) Estimate the frequency response of the amplifier that would be required to faithfully record the ECG shown, as well as any variation in signal with respiration?
c) What sources of electrical noise may occur over this frequency range in practice? What steps can be taken to minimize these?

4.3
Explain, with reference to ionising radiation interactions, how high energy radiation for use in radiotherapy is produced. Compare the dose deposition profile in tissue of photons and protons. Considering issues of normal tissue dose, why might you use protons instead of photons for radiotherapy?

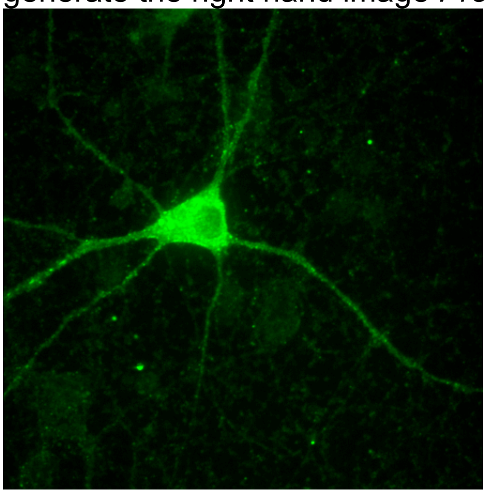
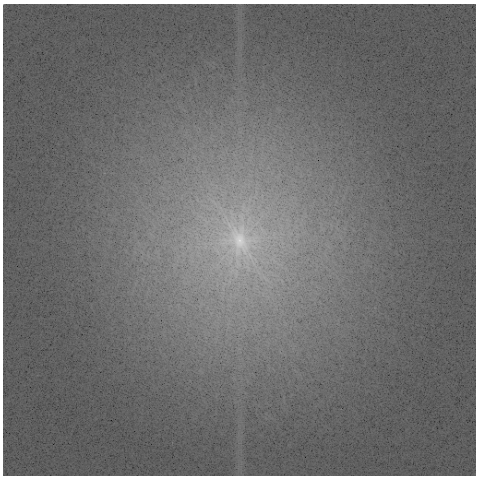
4.4
a) Explain why fractionation is used in radiotherapy.
b) Describe the need for a clinical target volume and planning target volume in radiotherapy. What is the difference between these volumes and what do they mean in a clinical context?
c) Many members of the public would not fully understand why brachytherapy is used in oncology, due to the perceived danger of radioactivity. Explain and justify why powerful gamma emitters such as Cs-137 should be used to treat patients. You should reference the ALARA principle in your answer.

Question 1.3 Image quality worked example

You will need to use the free software Fiji (Fiji Is Just ImageJ) to perform the calculations for this question. It can be downloaded for free from here: https://imagej.net/Fiji/Downloads

Follow the steps below to process and evaluate the image quality in two example images. Submit answers to the points labeled in the list below as "Question" to your supervisor.

Image content and spatial frequency

1. Open the sample image Neuron *File -> Open Samples -> Neuron*
2. Explore the image data profile of the different colour channels by moving your mouse around on the image and looking at the 'value' in the Fiji tool bar for each of the 5 channels.
3. Open the Brightness & Contrast tool using *Image -> Adjust -> Brightness/Contrast*. Look at how the image in the different channels changes as you adjust the thresholds. Reset the values after exploring
4. Separate out the stacked images into separate ones using *Image -> Colour -> Split channels*
5. Select a single image and look at the distribution of pixel data values in more detail using *Analyse -> Histogram*. Take the log, think about how this changes the visual appearance of the image.
6. Choose the image C2 (green). Perform a fast fourier transform to generate the right hand image *Process -> FFT -> FFT*
7.  
8. ***Question: What do the different parts of the FFT image correspond to in the original image?***
9. To answer this question, use the *Edit -> Fill* function to remove various parts (e.g. centre, edges, axes …) of the Fourier transformed image and then use *Process -> FFT -> Inverse FFT* to look at the result. You will need to perform multiple forward transforms to explore this in depth. Make sure to use the function *Image -> Rename* for each forward transform that you make so that you can keep track of your findings.
10. Draw profiles across the neuron using *Analyse -> Plot profile* and compare between different inverse transforms.
11. Now try to apply a band pass filter (*Process -> FFT -> Band pass filter*) to the same image with different settings.

12. ***Question: What have you learnt about the relationship between spatial dimensions (x,y,z) and spatial frequency dimensions (kx,ky,kz)?***

<u>*Resolution and signal-to-noise ratio*</u>

1. Open the microscopy image of embryos using *File -> Open Samples -> Embryos.*
2. Use *Analyse -> Set measurements* to choose the metrics that you would like to extract from the image. Default options are: area, min/max and mean. Think about what values you need to extract from your regions of interest to calculate contrast and signal-to-noise ratio
3. Draw regions of interest around an embryo and an equivalent sized region in the background, using *Analyse -> Measure* to extract your metrics. Think about why size of the region of interest is important.
4. ***Question: What is the available contrast and signal-to-noise ratio in this image?***
5. Now repeat this process in the image Blobs.
6. Apply in both images a blurring step *Process -> Filters -> Gaussian blur*.
7. ***Question: What do you observe about the contrast and signal-to-noise ratio having applied the blurring step? What does this tell you about the influence of spatial resolution on these parameters?***

Additional Problems

1
What are the main design developments and technological advances in CT scanners that have enabled a reduction in imaging time to be achieved?

2
Describe and explain how the CT number for a tissue might be expected to change with kVp.

3
In a certain ultrasound beam, the last axial maximum is 0.2 m from the face of the circular piezoelectric transducer and is continuously excited. If the speed of sound is 1500 ms^{-1} in tissue and the frequency of ultrasound is 3 MHz, what is:

a) The transducer diameter?
b) The angle of divergence in the far field?

4
Describe the different methods of measuring blood vessel concentration and perfusion in tissue by ultrasound. Which deposits the largest amount of energy in tissue? How can the potential adverse effects in tissue be minimised?

5
A Doppler ultrasound transducer emits ultrasound whose wavelength is 0.3mm. What is the frequency shift observed as the probe is angled at 45 degrees relative to a blood vessel with a flow of 0.1 ms^{-1}?

6
Outline the methodology for performing fMRI and diffusion weighted imaging. Give an example application of each method.

7
Imaging in nuclear medicine (using single photon emitters) can be performed using planar or tomographic geometry.

a) For each method, give an example of the clinical use and discuss why it is suitable for the application. You should make reference to the spatial and temporal resolution, signal to noise ratio and financial burden in your answer.
b) Comment on why bone scans represent the bulk of the workload for most Nuclear Medicine Departments.
c) Give an example of a nuclear medicine procedure where dynamic scanning is required. Suggest how the same procedure could be conducted with an imaging modality that uses non-ionising radiation and what the strengths and weaknesses of this approach would be.