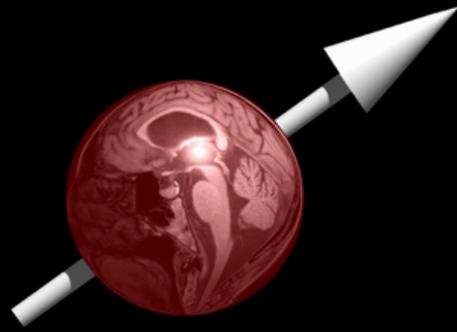


The Physics of MRI



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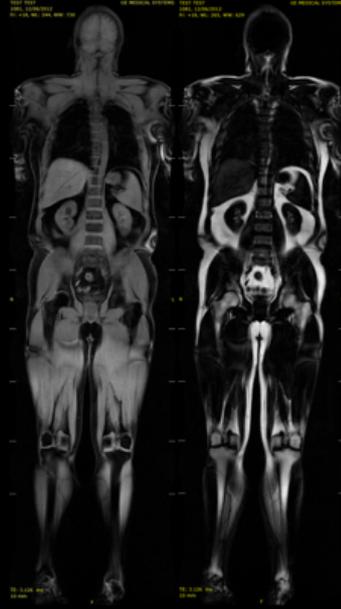
Cambridge University Hospitals 
NHS Foundation Trust

Learning Outcomes

- After these lectures you should be able to:
 - ◊ Explain how nuclear spin gives rise to magnetic resonance
 - ◊ Understand the principles of T_1 , T_2 and T_2^* relaxation
 - ◊ Explain the principles of MR image formation
 - ◊ Describe the spin echo and gradient echo pulse sequences
 - ◊ Outline the basic components of an MRI system
 - ◊ Understand the safety issues related to MRI
 - ◊ Describe some of the clinical applications of MRI

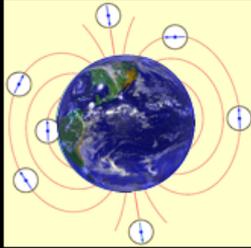
Magnetic Resonance Imaging

- Based upon nuclear magnetic resonance (NMR)
- MRI primarily looks at the **nucleus** of hydrogen
 - Human body 60% water and 16% fat
- MRI uses strong **magnetic** fields
 - Protons behave as microscopic bar magnets
- Protons **resonate** with external magnetic field
 - The net magnetisation can be manipulated



A typical adult is approximately 60% water, 16% fat, 18% proteins and 6% other elements. The majority of clinical MRI utilises the nucleus of hydrogen, i.e. a single proton, although other NMR active nuclei can be imaged in certain rare circumstances. Protons have a magnetic moment which can interact with the magnetic fields of the MRI systems to create images. The hydrogen protons can be considered to precess at a specific frequency and they can interact with a matching, i.e. resonant, external alternating magnetic field.

Magnetic Field Strength



0.00006 T
x 1



0.003 T
x 50



0.6 T
x 10,000



3.0 T
x 50,000



Nikola Tesla
(1856-1943)

1 tesla = 10,000 gauss



Carl Friedrich Gauss
(1777-1855)

We are all familiar with the fact that the earth has a magnetic field, that's the reason that compass needles point almost North. We measure magnetic field strength in units of Tesla, named after Nikola Tesla, the famous Serbian-American inventor. The Earth's magnetic field is approximately sixty millionths of a tesla, or sixty microtesla. Another hugely important use of magnets is for sticking plastic letter to the doors of refrigerators. These magnets may go up to around three thousandths of a tesla or three millitesla, and about fifty times stronger than the earth's magnetic field. Stronger magnets like those you might see in a junkyard for moving heavy iron objects may be around half a tesla or ten thousand times stronger than the earth's magnetic field. In order to generate such large magnetic fields requires the use of an electromagnet where the magnetic field is created by an electric current flowing through a wire wrapped a large number of times around an iron core that helps to increase the magnetic field. Finally, the magnets used in in the NHS for MRI are typically either 1.5T or 3.0T. This is a picture of the 3.0T MRI scanner that we have in the Radiology department here. This magnet is about fifty thousand times stronger than the earth's magnetic field.

Wheelchair flies across hospital room

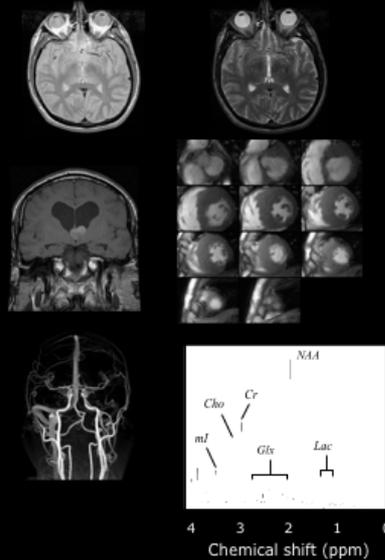
A hospital is facing a £20,000 bill to repair damage caused by a wheelchair which was flung across a room by powerful magnets in an MRI machine.



Fortunately, nobody was injured in this case but this wheelchair will have been attracted to the magnet at around 30 miles per hour. The field is generally never switched off in a superconducting magnet, which is why it cost £20,000 pounds to switch off the magnet, remove the wheelchair and power it up again, as well as repairing the damage to the covers surrounding the magnet.

MRI Advantages

- ▶ No ionising radiation
 - Magnetic fields only
- ▶ Multi-planar
 - Axial, coronal, sagittal, single/double oblique
- ▶ Multiple contrast mechanisms
 - Proton density, T_1 , T_2 , flow, diffusion, chemical exchange, spectroscopy ...
- ▶ Good spatial resolution
 - Variable field-of-view
- ▶ Reasonable temporal resolution
 - Dynamic/functional imaging



Unlike X-ray or nuclear medicine based imaging methods MRI does not utilise ionizing radiation. Instead it relies on magnetic fields for image formation. MRI is a tomographic imaging techniques that acquires data in multiple thin slices or sections. These sections can be acquired in any orientation, including oblique planes. MR images can reflect a number of different contrast mechanisms including proton density, T_1 and T_2 relaxation times and number of other microscopic and macroscopic phenomena. MRI is particularly noted for its excellent soft-tissue contrast. Spectroscopy can be used to investigate different tissue metabolites since the protons within these molecules exhibit small chemical shifts

A Brief History of NMR & MRI

- › Based on the principle of Nuclear Magnetic Resonance discovered by Bloch and Purcell *simultaneously* in 1946 (awarded 1952 Nobel Prize in Physics)
- › The initial concept for the medical application of NMR originated with Damadian in 1971
- › NMR imaging of two tubes of water first demonstrated by Lauterbur in 1973
- › Slice selection (and other things) invented by Mansfield in 1973
- › First commercial system developed by EMI in 1975
- › 2003 Nobel Prize for Physiology or Medicine award to Lauterbur and Mansfield for "their discoveries concerning magnetic resonance imaging"



Edward Purcell
(1912-1997)



Felix Bloch
(1901-1999)



Raymond Damadian
(1936-)



Peter Mansfield
(1933-2017)



Paul Lauterbur
(1929-2007)

In 1952 Edward Purcell (Harvard) and Felix Bloch (Stanford) jointly received the Nobel Prize for physics 'for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith'. Of Purcell's discovery, the Boston Herald reported that 'it wouldn't revolutionize industry or help the housewife'. Bloch, a Swiss-born Jew and friend of quantum physicist Werner Heisenberg, quit his post in Leipzig in 1933 in disgust at the Nazi's expulsion of German Jews (as a Swiss citizen, Bloch himself was exempt). Bloch's subsequent career at Stanford was crammed with major contributions to physics and he has been called 'the father of solid state physics'. The 2003 Nobel Prize for Physiology or Medicine was awarded to Paul Lauterbur and Sir Peter Mansfield for 'for their discoveries concerning magnetic resonance imaging'. Paul Lauterbur is said to have been inspired to use field gradients to produce an image whilst eating a hamburger. His seminal paper 'Image Formation by Induced Local Interactions. Examples Employing Nuclear Magnetic Resonance' (*Nature* **242**, March 16, 1973) was originally rejected. 30 years later, *Nature* placed this work in a book of the 21 most influential scientific papers of the 20th century. Damadian and his colleagues at the State University of New York, starved of mainstream research funding, went so far as to design and build their own superconducting magnet operating in their Brooklyn laboratory and the first human body image by NMR is attributed to them. There is some dispute about who actually is the founder of modern **M**agnetic **R**esonance **I**maging (MRI), but one thing is certain, Damadian coined the first MR acronym, namely **F**ONAR (**F**ield **f**ocused **N**uclear **m**agnetic **R**esonance).

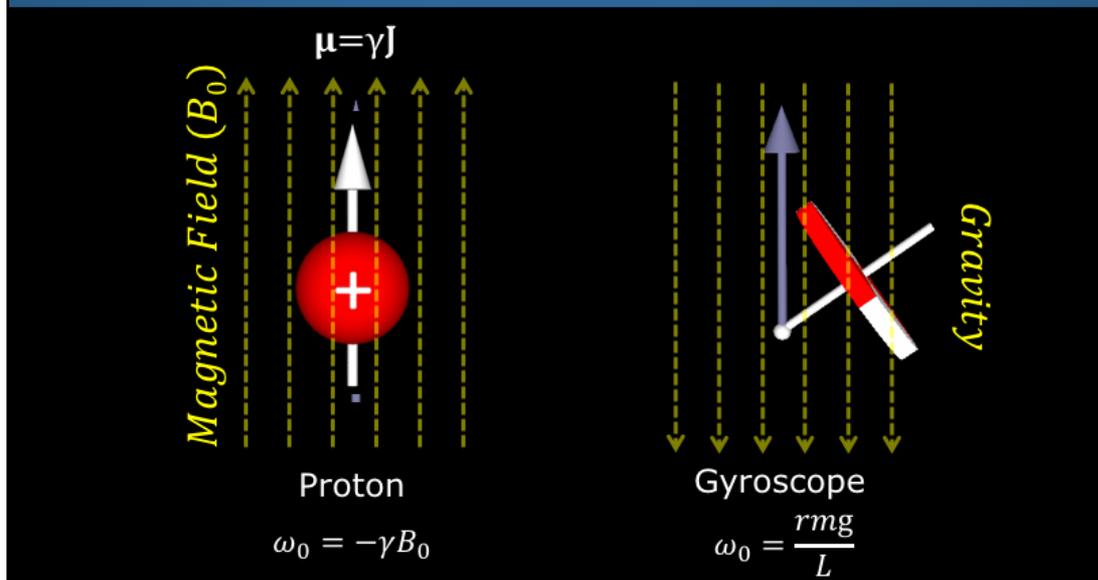
MRI Limitations

- Strong magnetic field
 - Safety, acoustic noise
- Relatively slow
 - c.f. ultrasound, CT, X-ray angiography
- Cannot directly visualise calcium
 - c.f. CT
- Complex technology
 - Expensive, requires skilled operators (Radiographers)
- Subjects can feel claustrophobic (bore diameter 60/70cm)



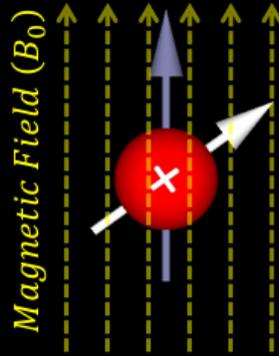
MRI has a number of limitations compared to other medical imaging techniques. Since it utilises magnetic field there are lethal safety concerns regarding the presence of ferromagnetic objects near MRI systems. Whilst not necessarily dangerous, the gradients used for spatial localisation can generate quite high acoustic noise levels. A typical MRI examination will comprise a number of imaging series, each having a different contrast mechanism and/or orientation. Whilst we can acquire some MRI images in under a second, most clinical imaging series take several minutes to acquire the data. A complete MRI examination can take anything from around 15 minutes to an hour depending upon the complexity of the investigation. Standard MRI sequences cannot visualise cortical bone, hence X-ray computerised tomography (CT) is still the method of choice for trauma imaging due to its speed and ability to visualise bone. The wide range of imaging parameters makes MRI a complex, expensive technology that requires skilled operators (Radiographers). The majority of MRI systems are based on superconducting magnet technology, with a 60-70cm patient aperture.

Spin



For the purposes of our discussion we will take a fanciful (and relativistically inadequate) representation of the proton as a spinning ball of charge. Due to the combination of a charge and spin we can consider that the proton behaves as a tiny bar magnet, i.e. it has a nuclear magnetic moment μ . The ratio of the magnetic moment μ to the angular momentum, J , is called the gyromagnetic ratio, γ . When placed in a static magnetic field (B_0) we can consider that it makes an angle with respect to the direction of B_0 . It therefore experiences a torque and precesses about B_0 at a characteristic frequency $\omega_0 = -\gamma B_0$. The minus sign which will get quietly dropped in future just means that defines a clockwise rotation about B_0 . The proton precession is analogous to the precession of a gyroscope in the Earth's gravitational field. Where r is the distance from the pivot, m is the mass, g is the acceleration due to gravity and L is the angular momentum.

Precessional Frequency

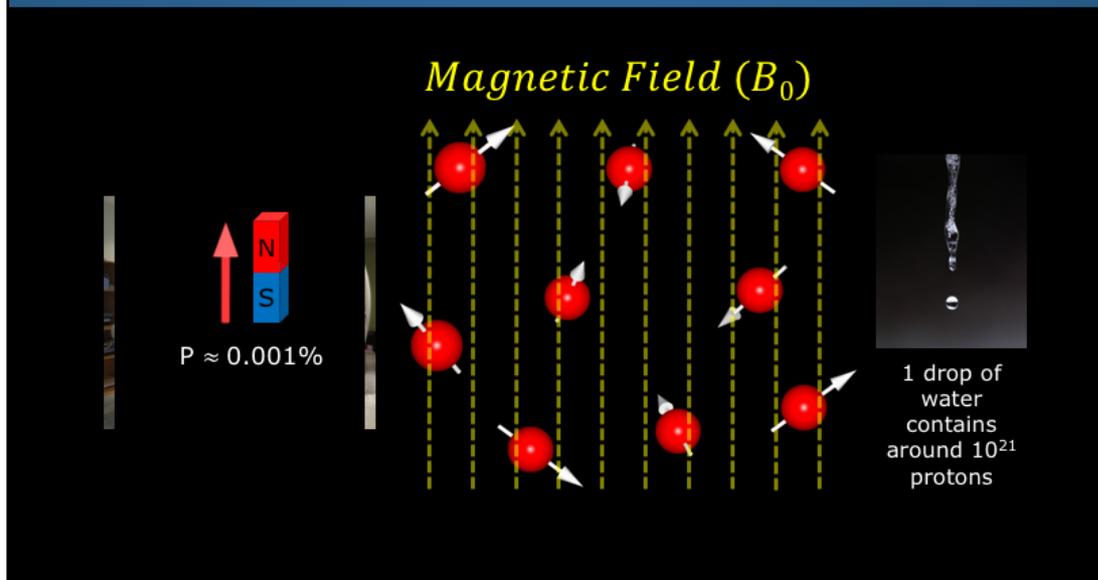


For ^1H $\gamma = 42.57\text{MHz/T}$
i.e., at 3.0T the precessional
frequency of ^1H is 128 MHz (RF)



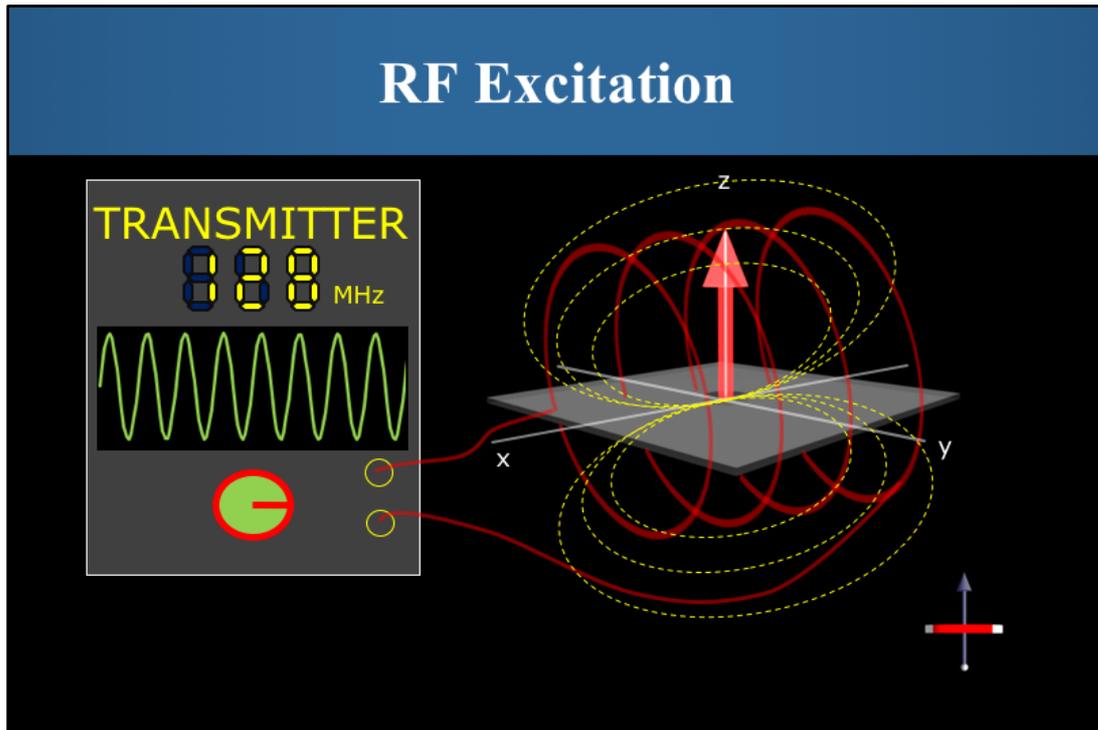
As we have seen the precessional frequency is dependent upon the strength of the magnetic field, and in our 3T magnet the protons in the body will precess at a frequency of 128MHz, i.e. in the radiofrequency part of the electromagnetic spectrum and a much lower frequency than those associated with X-rays for example. This is one of the reasons why MRI is actually a very safe imaging modality, unlike X-rays that involve the patient receiving a small dose of radiation.

Net Magnetization (Polarization)



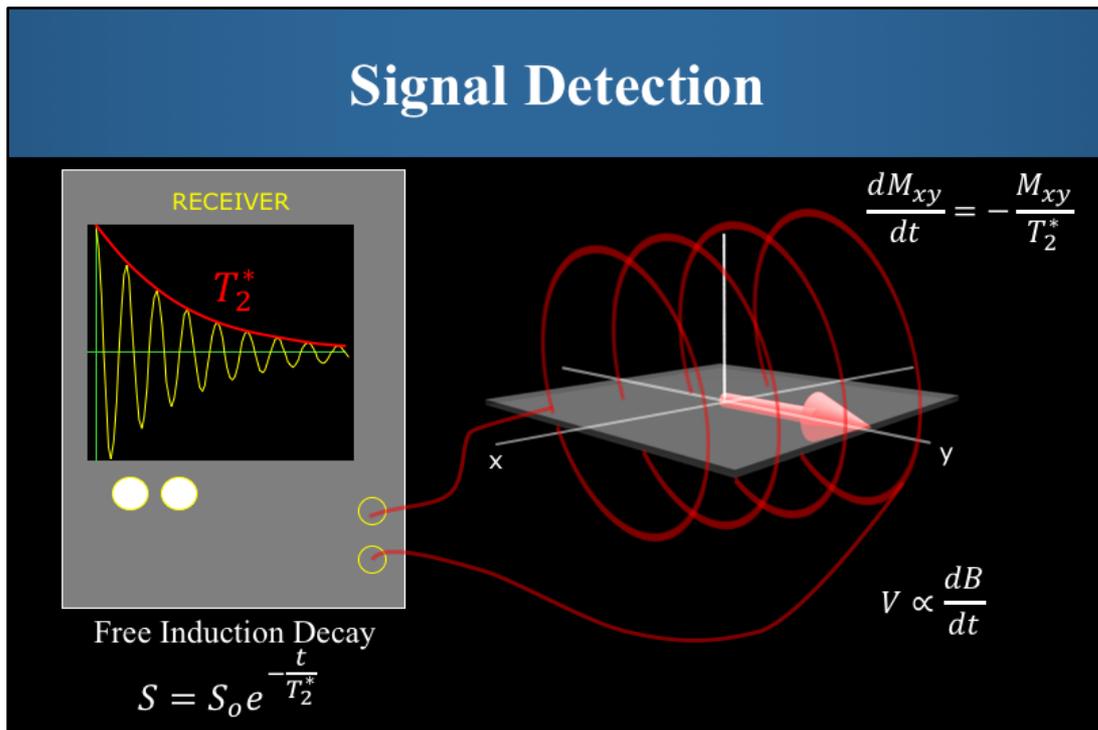
In addition to the individual protons precessing around the direction of the main magnetic field we can see that eventually just over half of the protons end up pointing in the same direction as the magnetic field and just under half end up pointing in the opposite direction. The ones pointing up appear like a large bar magnet pointing in the same direction as the magnetic field, whilst the one pointing down appear as a slightly smaller bar magnet pointing in the other direction. Since they are in opposite directions they subtract from each other, leaving a small, but measurable magnetisation pointing in the same direction as the magnetic field, this is known as the nuclear polarisation. Even so this net magnetisation is a very small percentage, fortunately there are a very large number of protons in the human body.

RF Excitation



Since the net magnetisation \mathbf{M} is pointing in the same direction as B_0 it is necessary to perturb the system away from equilibrium and then study how it returns back to equilibrium. We will denote the z-direction as the direction of the net magnetisation at equilibrium. We can perturb \mathbf{M} away from z by applying a second (weaker) magnetic field, called B_1 , at right angles to B_0 . We can create this alternating B_1 field by applying an alternating current to a coil, creating an alternating magnetic field inside the coil at right angles to B_0 . If B_1 is alternating at the same frequency as the natural precessional frequency, i.e. is resonant with (in this case at 3T we use 128MHz) then in the laboratory frame of reference \mathbf{M} will spiral down for as long as B_1 is applied. We often require that the magnetisation be tipped 90° away from z and into the transverse (x,y) plane. This is similar to continually “pushing” on a gyroscope with a force at right angles to gravity.

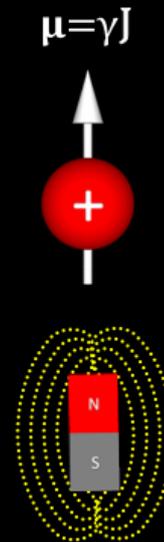
Signal Detection



Once \mathbf{M} has been tipped through 90° , if we turn off B_1 then \mathbf{M} will continue to precess in the transverse plane. This (weak) precessing, i.e. alternating magnetisation will then induce an emf in the same coil that was originally used to create the B_1 field. For various reasons \mathbf{M} will decay in amplitude whilst precessing. The induced emf can be seen on the oscilloscope as a exponentially decaying sinusoid. This is known as a free induction decay (FID) and is the most basic NMR signal. The time constant of the decay is characterised by a value known as T_2^* .

Principles of NMR

- ▶ Protons possess an intrinsic angular momentum \mathbf{J} and are positively charged
- ▶ A moving charge generates a magnetic field
- ▶ Hence we can consider a proton to behave like a bar magnet, i.e. it has a magnetic moment $\boldsymbol{\mu}$
- ▶ The magnetic moment $\boldsymbol{\mu}$ is given by $\boldsymbol{\mu}=\gamma\mathbf{J}$, where γ is the gyromagnetic ratio



The physical basis of Nuclear Magnetic Resonance (NMR) centres around the concept of a nuclear “spin,” its associated angular momentum and its magnetic moment. Spin is a purely quantum mechanical quantity with no classical analogy despite the simplified representation used here. For a hydrogen nucleus, i.e., a single proton (^1H) the spin angular momentum \mathbf{J} and the magnetic moment $\boldsymbol{\mu}$ are related through the proportionality constant γ , known as the gyromagnetic ratio and is a fundamental constant of a nucleus

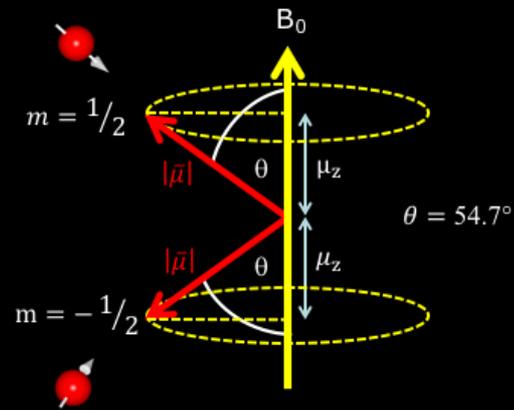
Nuclear Spin

- ▶ The magnitude and direction of J is characterized by the nuclear spin quantum number I
- ▶ Atoms with equal number of protons and neutrons have no spin
 - $I=0$, e.g., ^{12}C , ^{16}O
- ▶ If the number of neutrons **plus** the number of protons is odd then the nuclear has half-integer spin
 - $I=1/2$, e.g., ^1H , ^{19}F , ^{13}C , ^{31}P , ^{15}N
 - $I=3/2$, e.g., ^{23}Na , ^{37}Cl
- ▶ If the number of neutrons **and** the number of protons are both odd then the nuclear has half-integer spin
 - $I=1$, e.g., ^2H , ^{14}N

We primarily deal with the nuclei of hydrogen, i.e., a single proton which has a spin quantum number of $\frac{1}{2}$.

Nuclear Splitting in an External Magnetic Field (B_0)

- $\mu = \gamma J = \hbar \gamma \sqrt{I(I+1)}$
- $\mu_z = \gamma J_z = \hbar \gamma m_I$, where magnetic moment quantum number $m_I \in \{-I, \dots, I-1, I\}$ for a total of $2I+1$ values
- For nuclei with $I = 1/2$, e.g. ^1H there are two orientations with $m_I = 1/2$ and $-1/2$
- Zeeman observed spectral line splitting in an external magnetic field in 1896



Pieter Zeeman
1865-1943

In 1896 Pieter Zeeman observed the splitting of optical spectral lines in an external magnetic field, which we will call B_0 . Since then the splitting of energy levels proportional to an external magnetic field has been called the "Zeeman effect". We know from quantum mechanics that the angular momentum J is quantized. The total orbital angular momentum J is given by the nuclear spin quantum number I which is equal to $1/2$ for protons. The vectors z projection is given by the reduced Planck's constant ($\hbar = \frac{h}{2\pi}$), where $h = 6.63 \times 10^{-34} \text{ J s}$ times m_I and where m_I is the magnetic moment quantum number and has values of $-I \leq m_I \leq I$

So for nuclei with $I = \frac{1}{2}$

The angle of J is given by

$$|\bar{\mu}| = \hbar \gamma [I(I+1)]^{1/2}$$

$$\mu_z = \pm \frac{1}{2} \hbar \gamma$$

$$\text{The angle is therefore } \cos^{-1} \left(\frac{\mu_z}{|\bar{\mu}|} \right) = \frac{1/2}{\sqrt{\left(\frac{1}{2}\left(\frac{1}{2}+1\right)\right)}} = \frac{1/2}{\sqrt{3/4}} = 0.544 = 54.7^\circ$$

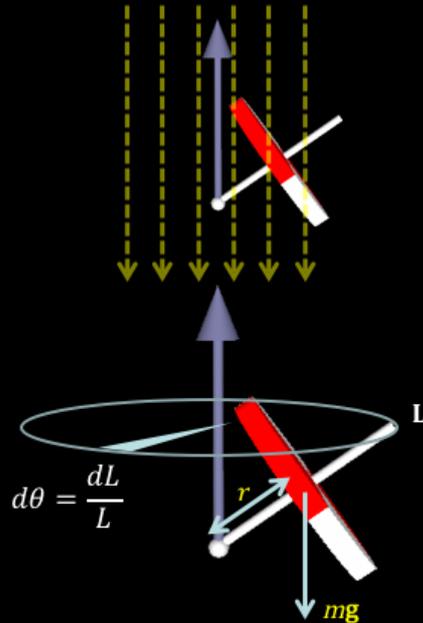
Gyroscope Precession

▸ Torque $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} = \mathbf{r} \times \mathbf{F}$

▸ $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} = \mathbf{r} \times m\mathbf{g}$, since $\mathbf{F} = m\mathbf{g}$

▸ $d\theta = \frac{dL}{L} = \frac{r m g}{L} dt$

▸ $\omega_0 = \frac{d\theta}{dt} = \frac{r m g}{L}$



A gyroscope, under the influence of gravity, will precess about the vertical.

Classical Description of Nuclear Precession

▸ Torque $\boldsymbol{\tau} = \frac{d\mathbf{J}}{dt} = \boldsymbol{\mu} \times \mathbf{B}$

▸ $\boldsymbol{\tau} = \frac{d\mathbf{J}}{dt} = \gamma \mathbf{J} \times \mathbf{B}$, since $\boldsymbol{\mu} = \gamma \mathbf{J}$

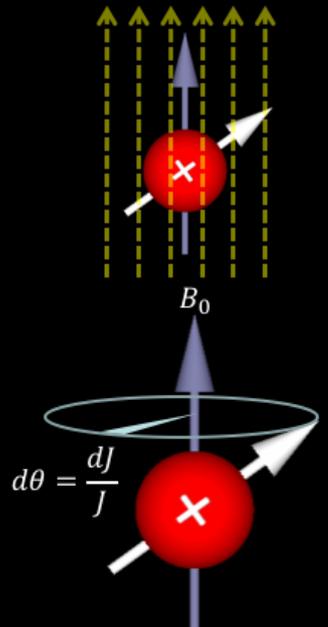
▸ $d\theta = \frac{dJ}{J} = \gamma B_0 dt$

▸ $\omega_0 = \frac{d\theta}{dt} = -\gamma B_0$

▸ $f_0 = \frac{\gamma}{2\pi} B_0$



Joseph Larmor
1857-1942



Similarly to the gyroscope the static magnetic field will cause our spinning nuclei to experience a torque $\boldsymbol{\tau}$ and hence precess around B_0 . This precessional frequency is given by the Larmor equation $\omega = \gamma B_0$. Larmor worked in an era when the basic structure of the atom was still being elucidated and quantum mechanics had yet to be developed. His famous equation of 1897 had nothing to do with NMR, as this phenomenon was still several decades away from being discovered. Instead it arose from attempts to explain the Zeeman splitting observed one year previously. Larmor's theory was that Zeeman's spectral lines were produced by charged particles (electrons) moving in elliptical orbits. Larmor demonstrated mathematically that these electrons should precess around the direction of the applied magnetic field. He further calculated that the frequency of precession was directly proportional to the strength of the applied field times a constant. Larmor's constant was directly related to the particle's charge/mass ratio. In later years as more became known about atomic structure, Larmor's equation was found to apply to any particle with spin or angular momentum, taking the form we recognize today. The Larmor frequency is a signed quantity and is negative for nuclei with a positive gyromagnetic ratio. This means that for such spins the precession frequency is negative, as shown in the animation. Note that conventionally, the Larmor equation is written as $\omega_0 = \gamma B_0$, where ω_0 is the angular frequency of the protons precession, i.e., $\gamma = 2.67 \times 10^8 \text{ radians s}^{-1} \text{ T}^{-1}$. ($\omega = 2\pi f$). However, I find this number unmemorable and angular frequencies are not as intuitively understandable as regular (scalar) frequencies. I will therefore often refer to γ as 42 MHz T^{-1} , i.e. $\gamma/2\pi$.

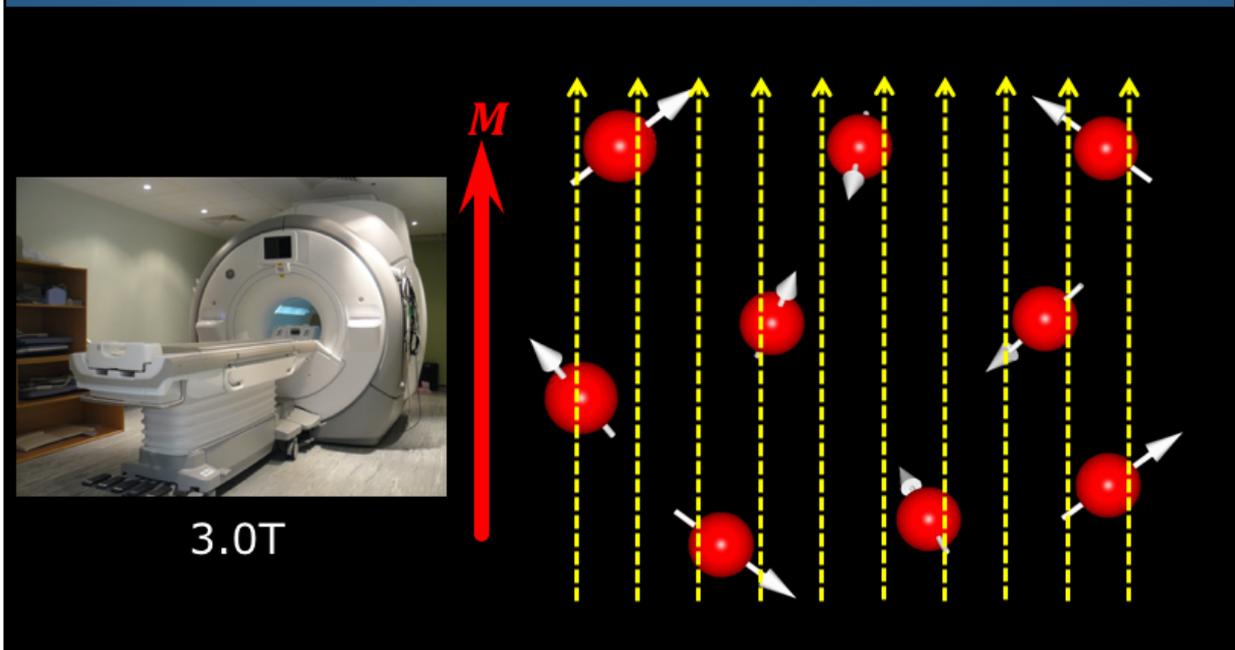
Some NMR Active Nuclei

Nucleus	Unpaired Protons	Unpaired Neutrons	Net Spin	γ (MHz/T)	Natural Abundance (%)
^1H	1	0	1/2	42.58	99.98
^{13}C	0	1	1/2	10.71	1.1
^{14}N	1	1	1	3.08	99.6
^{19}F	1	0	1/2	40.08	100
^{23}Na	1	2	3/2	11.27	100
^{31}P	1	0	1/2	17.25	100

The neutron doesn't have charge but has a magnetic moment. This can be reconciled with the classical model if the neutron is considered as a proton orbited by a negatively charged pion. The neutron has spin $1/2\hbar$ but no net charge. The neutron is composed of three quarks and the magnetic moments of these elementary particles combine to give the neutron its magnetic moment.

The gyromagnetic ratio γ dictates the precessional frequency of the nucleus, i.e. 42.57MHz/T. Therefore in a 3T MRI system the protons precess at approximately 128MHz.

Protons in a Magnetic Field



With no applied magnetic field, all nuclei (aka spins) are in the same energy state. Their magnetic moments are randomly oriented and do not form any coherent magnetization. However when the nuclei experience is placed in a static magnetic field (B_0) such as a 3.0T MRI system the nuclei split into two energy states, either aligned with "spin-up" or against "spin-down" the direction of B_0 . The "spin-up" state is slightly preferred and thus has a lower energy level. This slight difference (0.001%) results in an overall net magnetisation (M) aligned in the same direction as B_0 .

Energy Level Splitting

- ▶ The energy E of a magnetic moment ($\boldsymbol{\mu}$) in a magnetic field \mathbf{B} is given by

- $E = -\boldsymbol{\mu} \cdot \mathbf{B} = \hbar\gamma m_l \mathbf{B}$

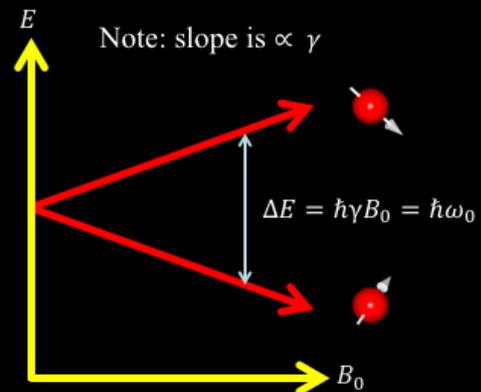
- ▶ Selection rules only allow for transitions between $-I \leq m_l \leq I$ so for $I = 1/2$

- $\Delta E = (1/2 - -1/2)\gamma\hbar B_0 = \hbar\gamma B_0$

- ▶ From De Broglie's wave equation

- $\Delta E = \hbar\omega$

- ▶ $\hbar\gamma B_0 = \hbar\omega_0$

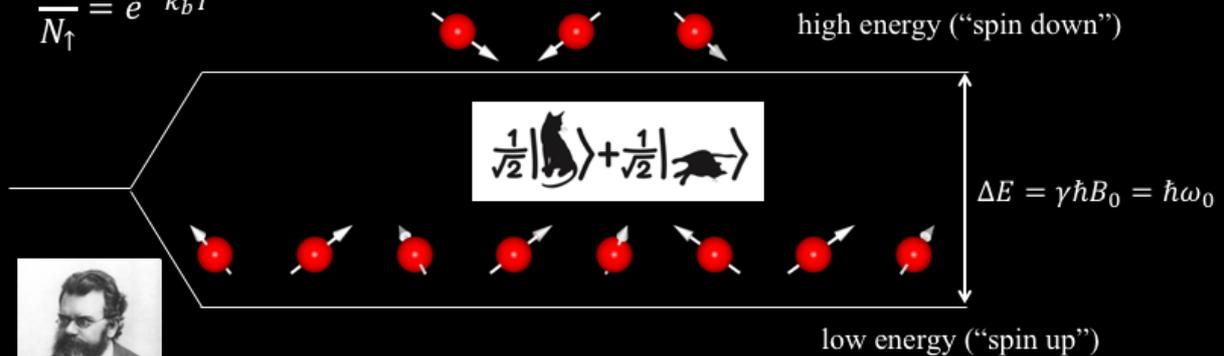


From a QM perspective the energy difference between the two orientations in a static magnetic field of B_0 is given by

Nuclear Polarisation

$$P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} \approx \frac{1}{2} \frac{\gamma \hbar B_0}{k_b T} \approx 10 \times 10^{-6} \quad (\text{At } 310\text{K and } 3.0\text{T})$$

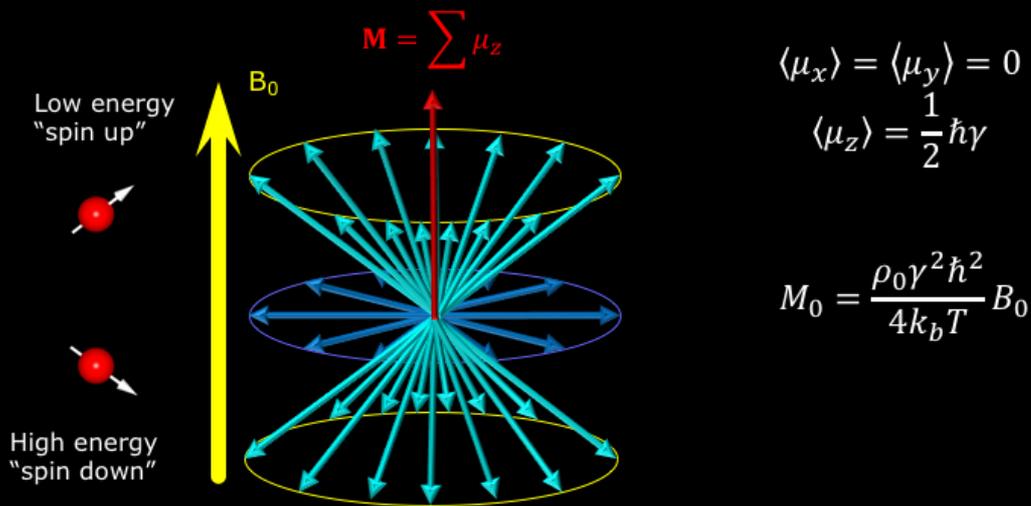
$$\frac{N_{\downarrow}}{N_{\uparrow}} = e^{-\frac{\Delta E}{k_b T}}$$



Ludwig Eduard Boltzmann
1844-1906

The population distribution, known as the polarisation (P), of the nuclei between the two states are characterised by the Boltzmann distribution which describes the distribution of energy among classical (distinguishable) particles. Where k_b is Boltzmann's constant (1.38×10^{-23} J/K) and T is the absolute temperature (human body temperature is normally about 310K). If we inject energy into this system at a angular frequency of ω_0 , we can induce transitions between the two energy states. A spin-up nuclei can absorb energy and transition to a spin down state and a spin-down nuclei can give up energy and transition to spin-up. The excitation must be at this specific frequency in order to "resonate" with the nuclei – this frequency selectivity is the origin of the term resonance in nuclear magnetic resonance. The cartoon of Schrodinger's cat is to remind you that the spin and magnetic moment exist in all directions simultaneously, but their average behaviour is non-zero in only one of the directions. Therefore these diagrams are an attempt to provide some additional insights into how NMR works but their limitations should be appreciated

Net Magnetisation



Water contains 6.67×10^{22} protons ml^{-1} . So at body temperature at 3.0T we get $M_0 \approx 0.02 \mu\text{Tml}^{-1}$. Assuming the human head has volume of 1500ml and is approximately 80% water we get $M_0 \approx 20 \mu\text{T}$

The spin (and associated magnetic moment and angular momentum) is probabilistic in nature (much in the same way that electrons surrounding the nucleus travel in probabilistic shells). Thus, each spin doesn't really align with B_0 , but rather exists in a probabilistic cone and spin-up and spin-down implies that the cone faces up or down. The spin and magnetic moment exist in all directions simultaneously, but the average behaviour is non-zero in only one of the directions. Each spin contributes $\frac{1}{2}$ to the overall z-magnetisation of the sample. We cannot observe individual spins, only the ensemble average \mathbf{M} .

MR Signal

- To create an MR signal we need to **excite** the spins out of equilibrium, i.e. we must tip the net magnetization away from the B_0 direction
- A **transmit coil** will create an orthogonal magnetic field (B_1), alternating at the Larmor frequency that will rotate the net magnetization into the transverse (x-y) plane (**RF pulse**)
- The precessing transverse magnetisation will induce a voltage in a **receiver coil**
- After the pulse the magnetization will return to thermal equilibrium by processes known as **relaxation**

The Bloch Equation

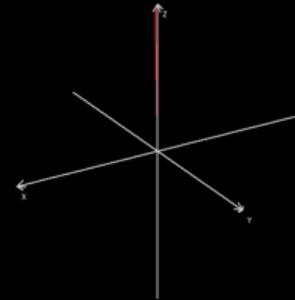
- ▶ The dynamics of nuclear magnetization are described phenomenologically by the Bloch equation

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma\mathbf{B} - \frac{M_x i + M_y j}{T_2} - \frac{(M_z - M_0)k}{T_1} + D\nabla^2 M$$

- ▶ The last term accounts for the transfer of magnetisation by diffusion, with D being the molecular self-diffusion coefficient
- ▶ The transverse component will dephase in the x-y plane with a time constant T_2 . This is known as “spin-spin”, or “transverse” or “ T_2 ” relaxation. **This process only involves energy exchange**
- ▶ The longitudinal component will regrow along the z-direction with a time constant T_1 . This is known as “spin-lattice”, or “longitudinal” or “ T_1 ” relaxation. **This process involve energy loss as heat to the surrounding macromolecules aka “lattice”**



Felix Bloch
1901-1999



Motion in the Presence of an Applied Field

- Net nuclear magnetisation \mathbf{M} will precess about \mathbf{B} at a frequency $\omega_0 = \gamma B_0$ the motion is given by

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma \mathbf{B}$$

- Define $\mathbf{M} = iM_x + jM_y + kM_z$ and expressing the cross product as the formal determinant

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B} = \gamma \det \begin{bmatrix} i & j & k \\ M_x & M_y & M_z \\ B_x & B_y & B_z \end{bmatrix}$$

- Using Sarrus's rule

$$\begin{aligned} \frac{d\mathbf{M}}{dt} &= \gamma \left(i(M_y B_z - M_z B_y) + j(M_z B_x - M_x B_z) \right. \\ &\quad \left. + k(M_x B_y - M_y B_x) \right) \end{aligned}$$

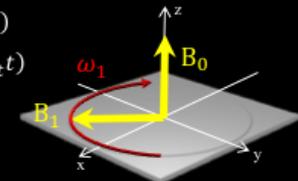
- In addition to the static field \mathbf{B}_0 we introduce an additional oscillating magnetic field \mathbf{B}_1 that rotates in the x-y plane at frequency ω_{rot}

$$B_1(t) = iB_1(t) \cos(\omega_{rot}t) - jB_1(t) \sin \omega_{rot}t$$

$$B_{1,x}(t) = B_1(t) \cos(\omega_{rot}t)$$

$$B_{1,y}(t) = -B_1(t) \sin(\omega_{rot}t)$$

$$B_{1,z}(t) = 0$$



$$\frac{dM_x}{dt} = \gamma(M_y B_0 + M_z B_1 \sin(\omega_{rot}t))$$

$$\frac{dM_y}{dt} = \gamma(M_z B_1 \cos(\omega_{rot}t) - M_x B_0)$$

$$\frac{dM_z}{dt} = \gamma(-M_x B_1 \sin(\omega_{rot}t) - M_y B_1 \cos(\omega_{rot}t))$$

Rotating Frame of Reference

- Let's define a rotating frame of reference with Ω as the rotational angular velocity vector

$$\Omega = -\omega_{rot}k$$

- With unit vectors in the rotating frame i', j', k' the net magnetisation \mathbf{M} becomes

$$\mathbf{M} = i'M_x + j'M_y + k'M_z$$

- The total time derivative of \mathbf{M} is

$$\frac{d\mathbf{M}}{dt} = \frac{dM_x}{dt}i' + M_x \frac{\partial i'}{\partial t} + \frac{dM_y}{dt}j' + M_y \frac{\partial j'}{\partial t} + \frac{dM_z}{dt}k' + M_z \frac{\partial k'}{\partial t}$$

- Note that $\frac{\partial u'}{\partial t} = \Omega \times u'$ where u' represents each unit vector

$$\left(\frac{d\mathbf{M}}{dt}\right)_{lab} = \frac{\partial \mathbf{M}}{\partial t} + \Omega \times (M_x i' + M_y j' + M_z k')$$

$$\left(\frac{d\mathbf{M}}{dt}\right)_{lab} = \left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rot} + \Omega \times \mathbf{M}$$

- Since we know the LHS is equal to $\mathbf{M} \times \gamma \mathbf{B}$

$$\mathbf{M} \times \gamma \mathbf{B} = \left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rotating} + \Omega \times \mathbf{M}$$

$$\left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rot} = \mathbf{M} \times \gamma \mathbf{B} - \Omega \times \mathbf{M}$$

$$\left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rot} = \mathbf{M} \times \gamma \mathbf{B} + \gamma \mathbf{M} \times \frac{\Omega}{\gamma}$$

$$\left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rot} = \mathbf{M} \times \gamma \left(\mathbf{B} + \frac{\Omega}{\gamma}\right)$$

$$\left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rot} = \mathbf{M} \times \gamma \left(\mathbf{B} - \frac{\omega_{rot}}{\gamma}\right) = \mathbf{M} \times \gamma \mathbf{B}_{eff}$$

The term $\left(\mathbf{B} - \frac{\omega_{rot}}{\gamma}\right)$ is often known as the effective field

Behaviour of \mathbf{M} in the presence of \mathbf{B}_1

- ▶ If $\omega_{rot} = \omega_0 = \gamma B_0$

$$\left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rot} = \gamma \mathbf{M} \times \left(\mathbf{B}_0 - \frac{\gamma B_0}{\gamma}\right) = 0$$

i.e. \mathbf{M} is static in the rotating frame

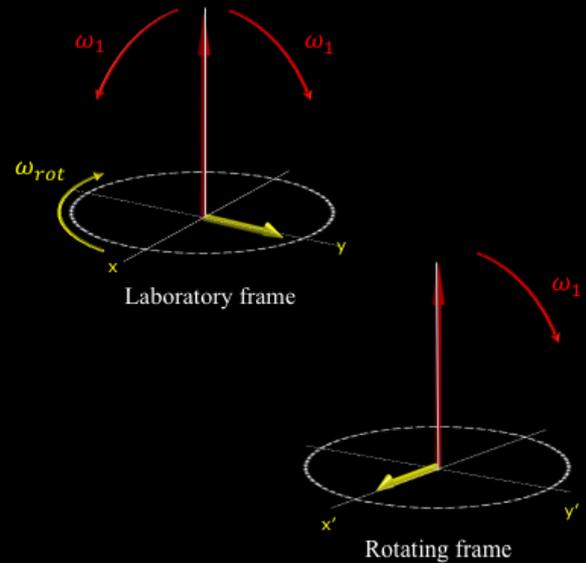
- ▶ If we include \mathbf{B}_1 , the effective \mathbf{B}_{eff} field in a frame rotating at ω_{rot} is given by

$$\mathbf{B}_{eff} = \left(\mathbf{B}_0 - \frac{\omega_{rot}}{\gamma}\right) \hat{k}' + \mathbf{B}_1 \hat{i}'$$

$$\left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rot} = \gamma \mathbf{M} \times \left(\mathbf{B}_0 - \frac{\omega_{rot}}{\gamma} + \mathbf{B}_1\right)$$

- ▶ If $\omega_{rot} = \omega_0$ then \mathbf{B}_1 is static and \mathbf{M} rotates about \mathbf{B}_1 at a frequency of $\omega_1 = \gamma B_1$

$$\left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rot} = \gamma \mathbf{M} \times \mathbf{B}_1$$



In the laboratory frame the magnetisation will spiral down about the effective field. In the rotating frame of reference the net magnetisation rotates about the direction of \mathbf{B}_1 at a frequency of $\omega_1 = \gamma B_1$. A RF pulse of duration t_{RF} will therefore tip the nuclei by an angle of $\alpha = \omega_1 t_{RF}$

Behaviour of \mathbf{M} in the presence of \mathbf{B}_1

- ▶ If $\omega_{rot} = \omega_0 = \gamma B_0$

$$\left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rot} = \gamma \mathbf{M} \times \left(\mathbf{B}_0 - \frac{\gamma B_0}{\gamma}\right) = 0$$

i.e. \mathbf{M} is static in the rotating frame

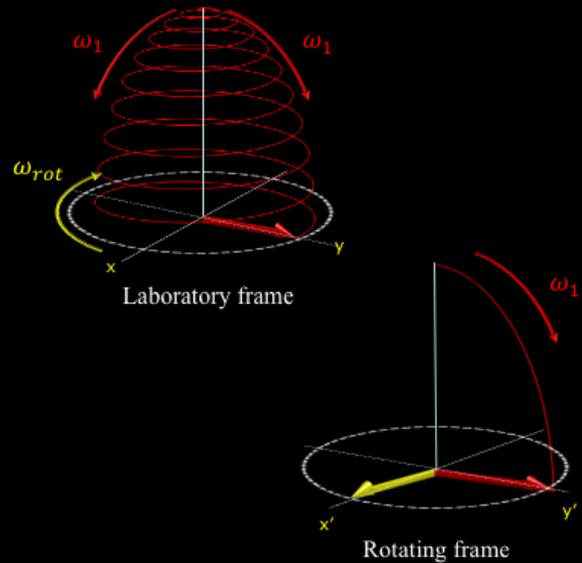
- ▶ If we include \mathbf{B}_1 , the effective \mathbf{B}_{eff} field in a frame rotating at ω_{rot} is given by

$$\mathbf{B}_{eff} = \left(\mathbf{B}_0 - \frac{\omega_{rot}}{\gamma}\right) \mathbf{k}' + \mathbf{B}_1 \mathbf{i}'$$

$$\left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rot} = \gamma \mathbf{M} \times \left(\mathbf{B}_0 - \frac{\omega_{rot}}{\gamma} + \mathbf{B}_1\right)$$

- ▶ If $\omega_{rot} = \omega_0$ then \mathbf{B}_1 is static and \mathbf{M} rotates about \mathbf{B}_1 at a frequency of $\omega_1 = \gamma B_1$

$$\left(\frac{\partial \mathbf{M}}{\partial t}\right)_{rot} = \gamma \mathbf{M} \times \mathbf{B}_1$$



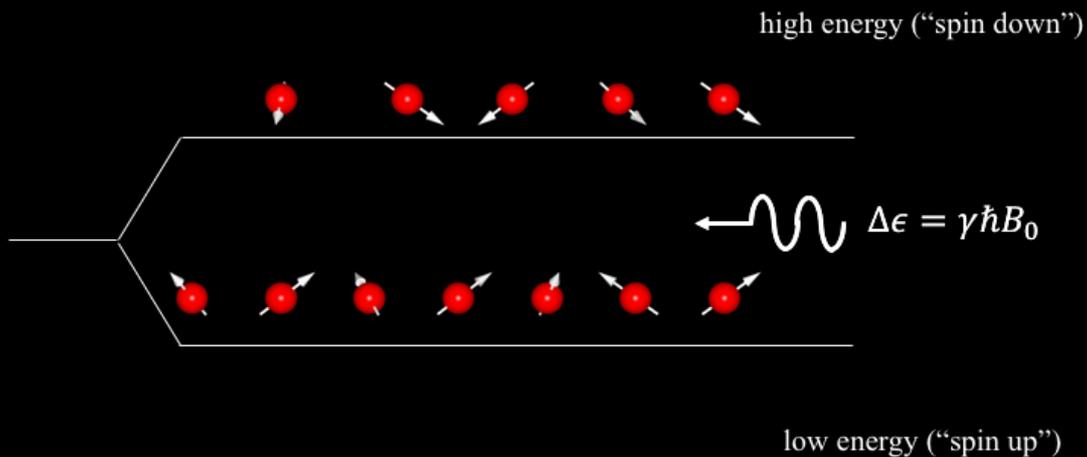
RF Excitation

- ▶ An alternating magnetic field of amplitude B_1 (μT) applied for time t_{RF} at the Larmor frequency ω_0 will tip \mathbf{M} by $\alpha = \omega_1 t = \gamma B_1 t_{RF}$

E.g. a rectangular (hard) RF pulse of 1ms duration with a B_1 of $5.87\mu\text{T}$ will flip the magnetization by 90° . Note that B_1 is considerably smaller than B_0

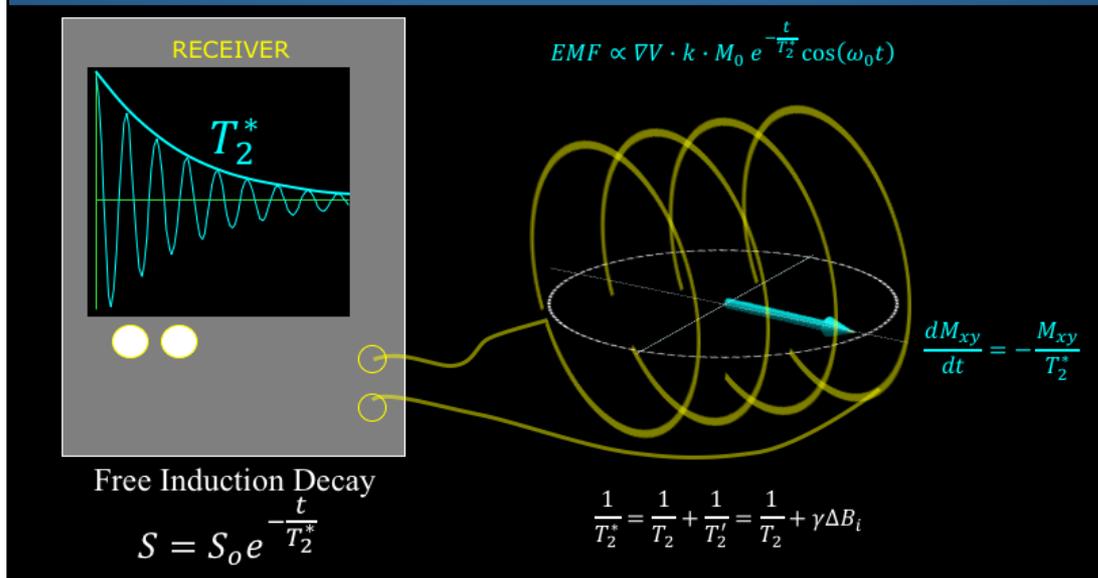
- ▶ RF pulses are the only way to rotate magnetization. Different flip angles are used for different applications
- ▶ In practice RF pulses will be shaped so more generally $\alpha = \gamma \int_0^{t_{RF}} B_1 dt$
- ▶ Note that MRI uses alternating magnetic fields in the RF part of the EM spectrum. It does not use radiowaves!

Excitation: Quantum Approach



Proceeding extremely cautiously with our QM approach. When the population of protons is irradiated by an RF field, protons can flip between energy levels. Spin-up protons can absorb energy to jump into the spin-down position, while those in the spin-down state are stimulated into giving up an equal amount of energy to drop into the spin-up state, and there is an equal probability of each transition. Since in equilibrium there are more spin-up protons than spin-down, the net effect will be absorption of energy from the RF wave, causing the "temperature" of the spin system to rise. The protons' temperature is considered separately from the temperature of the surrounding tissues, known as the lattice, which will eventually come into equilibrium with the spins. We will come back to this idea when we consider spin-lattice relaxation. Taking the simple idea of population difference and absorption of RF, it can be seen that the maximum absorption will be when all the spin-down protons have flipped into the spin-up position and vice versa. This is known as population inversion, and can be easily considered as a 180° pulse which flips the magnetization from z to $-z$. The definition of a 90° pulse can then be considered to be half that amount of energy, which is thought of as equalizing the populations leaving no magnetization along the z axis. Thus far the QM concepts seem to agree with our macroscopic observations, and they can be helpful up to this point.

Magnetization Relaxation & Signal Detection

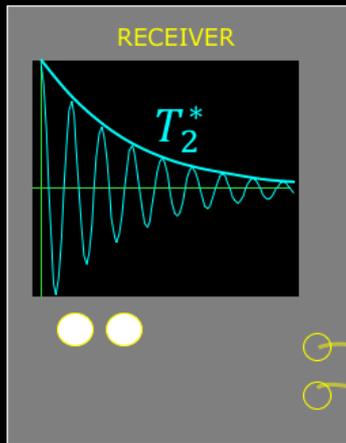


Relaxation is expressed in the Bloch equation. We have already met T_2 relaxation which is caused by the irreversible interaction of individual spins. This means that the individual nuclei that make up the net magnetisation will start to precess at different rates in the transverse plane due to spin-spin (T_2) interactions as well as other nonuniformities in the static magnetic field, either intrinsic to the magnet or due to magnetic susceptibilities within the body, these have a time constant given by T_2' . The overall relaxation time is the sum of these relaxivities, i.e.,

In the laboratory

frame we can see some nuclei precessing faster (yellow) and some precessing slower (white) than the nominal Larmor frequency. The cyan arrow represents the net magnetisation so it decreases in amplitude as the nuclei dephase with respect to each other in the transverse plane. The induced voltage is known as the free induction decay (FID)

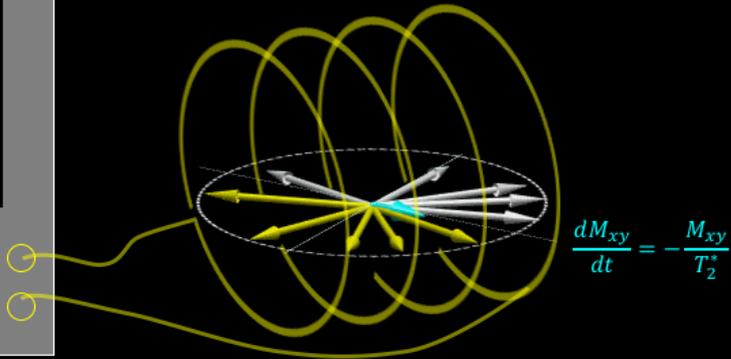
Magnetization Relaxation & Signal Detection



Free Induction Decay

$$S = S_0 e^{-\frac{t}{T_2^*}}$$

$$EMF \propto \nabla V \cdot k \cdot M_0 e^{-\frac{t}{T_2^*}} \cos(\omega_0 t)$$



$$\frac{dM_{xy}}{dt} = -\frac{M_{xy}}{T_2^*}$$

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'} = \frac{1}{T_2} + \gamma \Delta B_i$$

Bloch Equations with Relaxation

- Following the RF excitation the transverse magnetisation decays to zero with a time constant T_2

$$\frac{dM_x(t)}{dt} = \gamma(M_y B_0 + M_z B_1 \sin(\omega_{rot} t)) - \frac{M_x}{T_2}$$

$$\frac{dM_y(t)}{dt} = \gamma(M_z B_1 \cos(\omega_{rot} t) - M_x B_0) - \frac{M_y}{T_2}$$

- And the longitudinal magnetisation returns to equilibrium with a time constant of T_1

$$\frac{dM_z(t)}{dt} = \gamma(-M_x B_1 \sin(\omega_{rot} t) - M_y B_1 \cos(\omega_{rot} t)) - \frac{(M_z - M_0)}{T_1}$$

- Note that at equilibrium $M_z = M_0$

- Expressing the magnetisation in complex form $M_{xy} = M_x + iM_y$ then in the rotating frame of reference where $\omega_0 = \omega_{rot}$ the equations simplify to

$$\frac{dM'_{xy}(t)}{dt} = -\frac{M'_{xy}}{T_2}$$

$$\frac{dM_z(t)}{dt} = -\frac{(M_z - M_0)}{T_1}$$

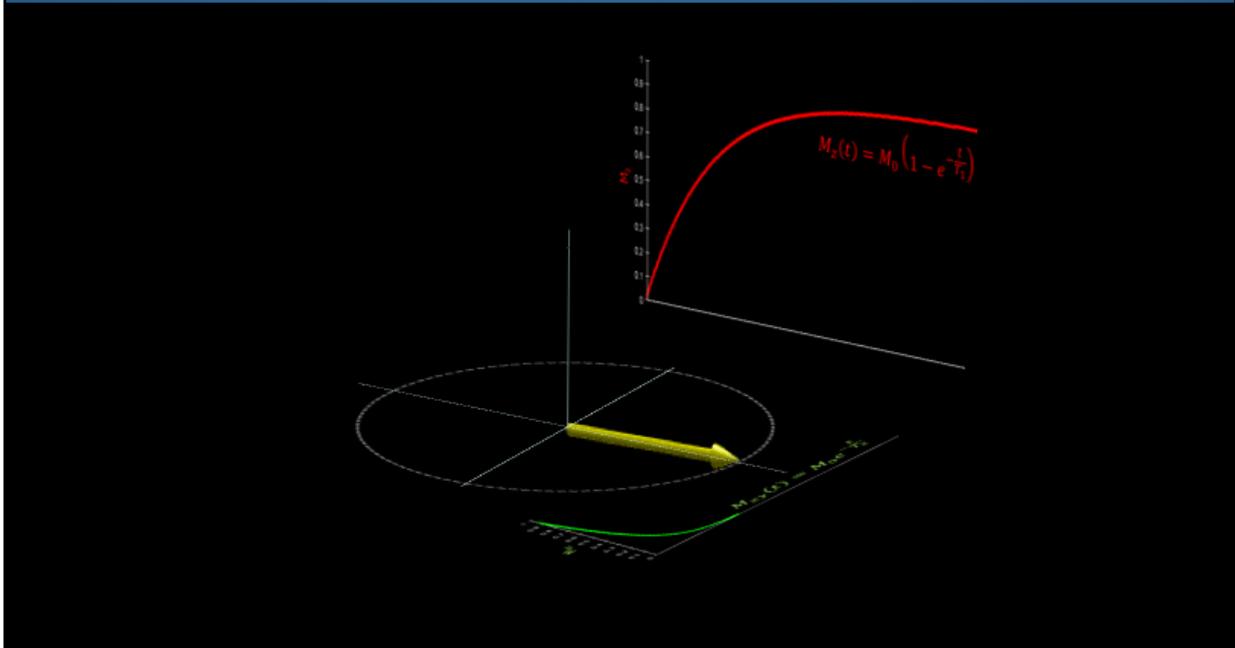
- These linear ordinary differential equations have the following solutions

$$M'_{xy}(t) = M_0 e^{-\frac{t}{T_2}}$$

$$M_z(t) = M_0 \left(1 - e^{-\frac{t}{T_1}}\right)$$

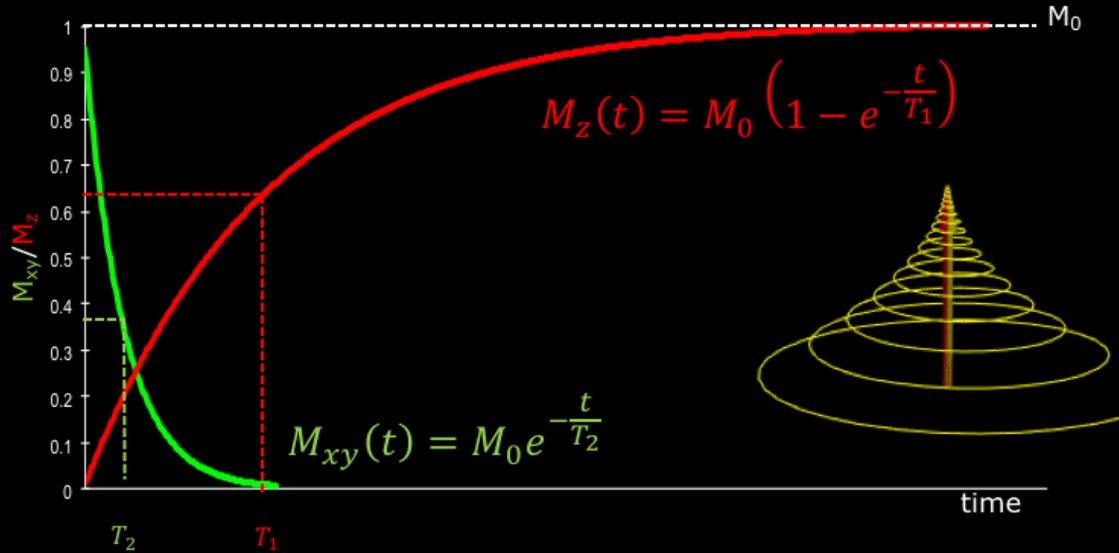
Solving the Bloch equation which treats the relaxation as a first order process gives the behavior of the transverse and longitudinal magnetization

Transverse and Longitudinal Relaxation



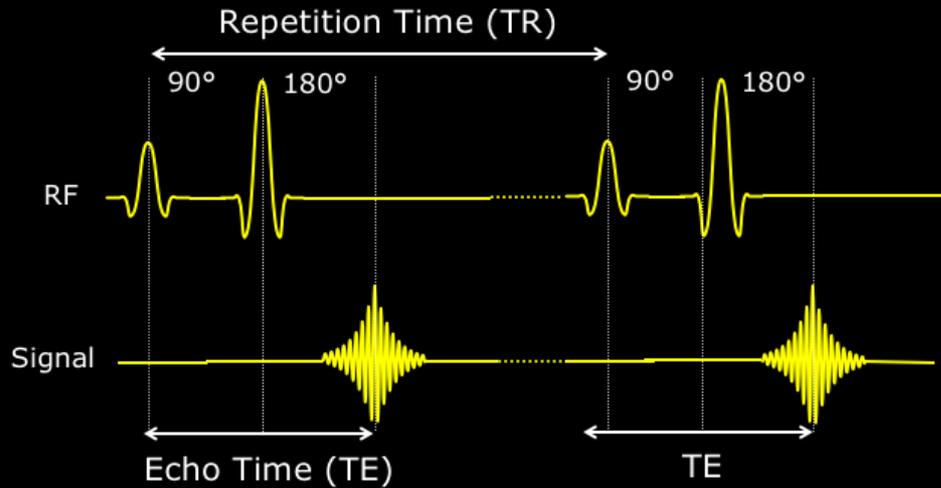
This figure shows the longitudinal magnetization recovery (red) and transverse magnetization decay (green) in the laboratory frame. In biological tissue T_2 is approximately an order of magnitude shorter than T_1 , i.e. For brain white matter the T_1 is 780ms and T_2 is 90ms. Remember that M_0 is proportional to the proton density ρ .

Relaxation



This figure shows the longitudinal magnetization recovery (red) and transverse magnetization decay (green). The T_1 is the time for 63% of the magnetisation to have recovered, whilst the T_2 is the time for 37% of the magnetisation to have decayed.

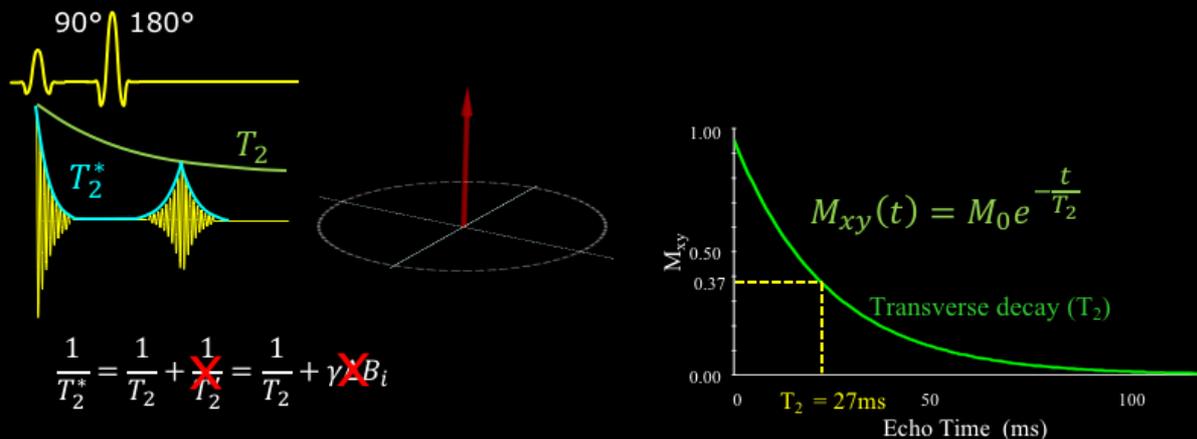
Spin Echo



The spin echo uses a 180° “refocusing” pulse to eliminate the dephasing caused by . Following a 90° excitation pulse the transverse magnetisation dephases. At a time τ later a 180° pulse flips the magnetisation in the transverse plane. After a further time τ the magnetisation is refocused as a spin echo. The echo time $TE = 2\tau$. Dephasing due to static T_2' effects are reversed, however T_2 related dephasing, which is irreversible, results in a reduced signal. Since different tissues have different T_2 values this will give rise to the desired T2-weighted image contrast. The longer TE the greater the T_2 signal differences. Note however, that as you increase TE the overall signal levels will decrease, i.e. contrast may improve but SNR will decrease.

T₂ Relaxation

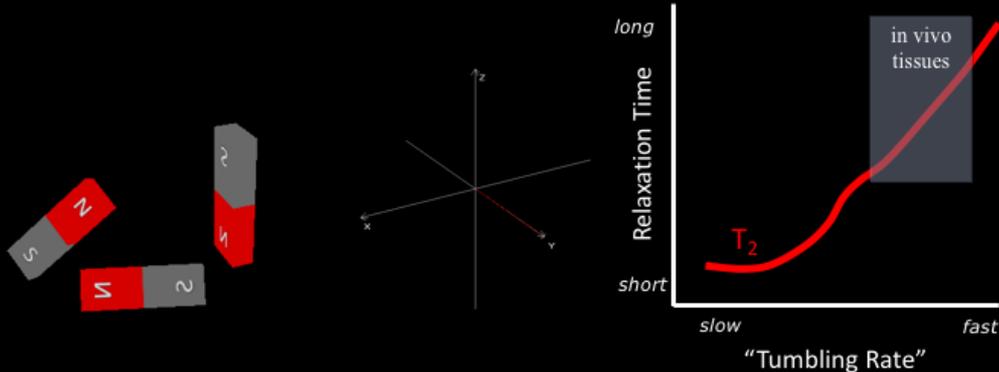
T₂' is eliminated by using a spin-echo (90°-180°) pulse sequence



Previously we showed the decay of the transverse magnetisation to be governed by T_2^* relaxation. In order to eliminate the often unwanted effects of T_2' we use a spin echo pulse sequence to eliminate the effects of static field non-uniformities and other magnetic susceptibility effects within the body. The spin echo sequences comprises a 90° pulse that tips the magnetisation into the transverse plane, followed by a 180° refocusing pulse. After the 90° pulse the magnetisation decays due to T_2^* relaxation. If after a certain time τ a 180° pulse is applied the disk of dephased magnetisation will undergo a 180° phase flip. Since the spins now have a reversed phase. If we wait for a further time τ the dephasing caused by the static magnetic field non-uniformities will be reversed, i.e. eliminated and an echo will be formed. Since spin-spin interactions are irreversible the echo will represent T_2 decay and hence the images will have T_2 -weighted contrast. Note that the time from the excitation pulse to the echo signal is called the echo time (TE) and $TE = 2\tau$.

T₂ Relaxation Mechanism

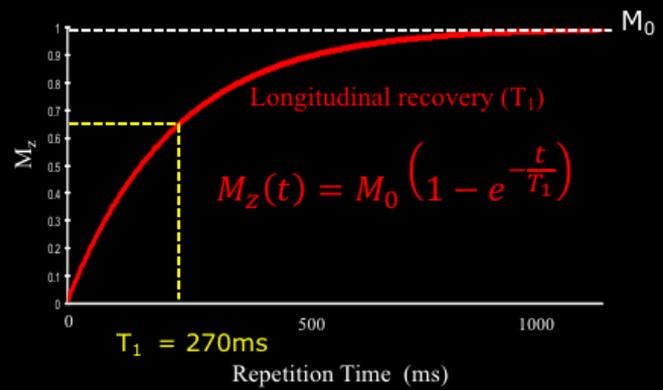
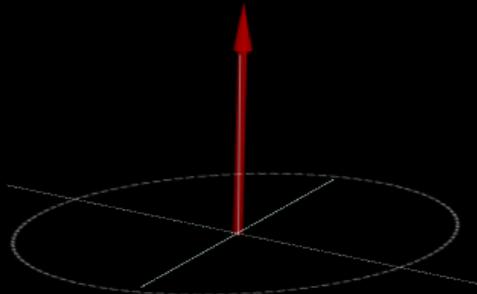
Magnetic moments of neighbouring protons alter local magnetic field causing a loss of phase coherence (spin-spin relaxation)



We know that T₂ relaxation arises from the exchange of energy between spins, hence the term 'spin-spin relaxation'. No energy is actually lost from the spin system but the decay of transverse magnetization arises from the loss of phase coherence between spins, which arises from magnetic field nonuniformities. These inhomogeneities may be either intrinsic or extrinsic, i.e. internal to the proton system or external in the scanner. Only the intrinsic inhomogeneities contribute to T₂. A description of molecular motions can also be used to describe the mechanism of T₂ relaxation. When molecules are tumbling very rapidly, i.e. we can consider them "free" and not "bound" to surrounding macromolecules then a particular spin dipole, i.e. bar magnet will see the local magnetic field as fluctuating very rapidly and effectively averaging out over a few milliseconds. This results in a relatively uniform local field and little dephasing, and is sometimes termed 'motional averaging'. Conversely a slowly tumbling molecule (bound protons close to large molecules) will see a relatively static magnetic field nonuniformity and will be more effectively dephased, i.e. will have a shorter T₂ with respect to other protons.

T₁ Relaxation

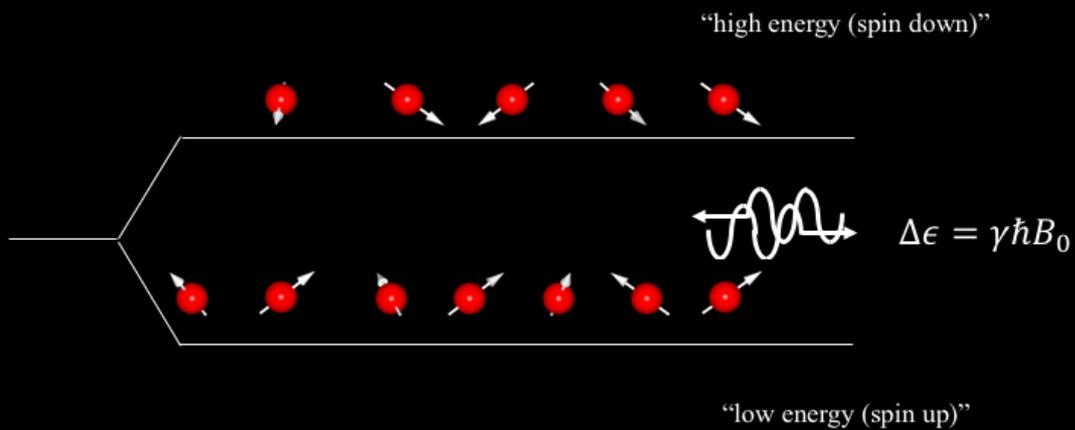
Magnetisation returns exponentially to thermal equilibrium



Simultaneously with the T₂ decay the magnetisation will recover back in the z-direction with a time constant given by the T₁ relaxation time. In a pulse sequence this recovery time is known as the repetition time (TR).

T₁ Relaxation: Quantum Approach

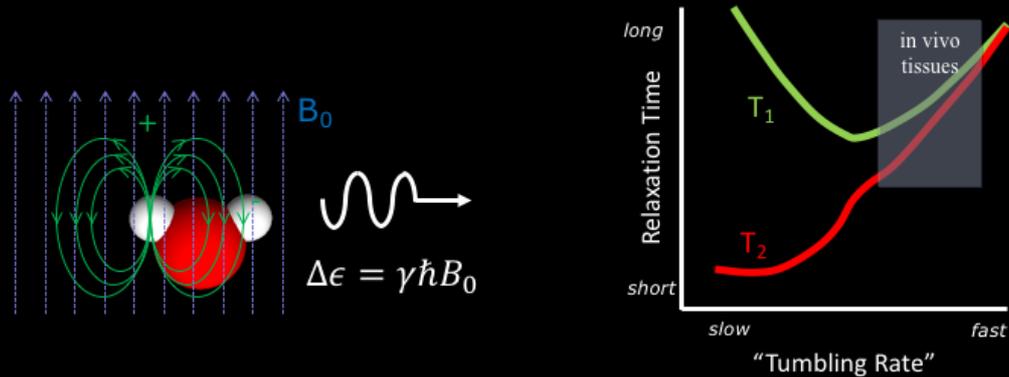
Quantum of energy (ΔE) stimulates move from high to low energy state, losing energy to surrounding lattice



We know that an RF pulse on average promotes protons from the low energy state to the high energy state causing a net absorption of energy. T₁ relaxation is the loss of the extra energy from the spin system to the surrounding environment, or 'lattice' (hence 'spin-lattice' relaxation time). However, the high energy state is a stable position for the proton and it does not return to the lower state spontaneously but requires an external stimulating field. Since the external B_1 field has been switched off, where does this field come from?

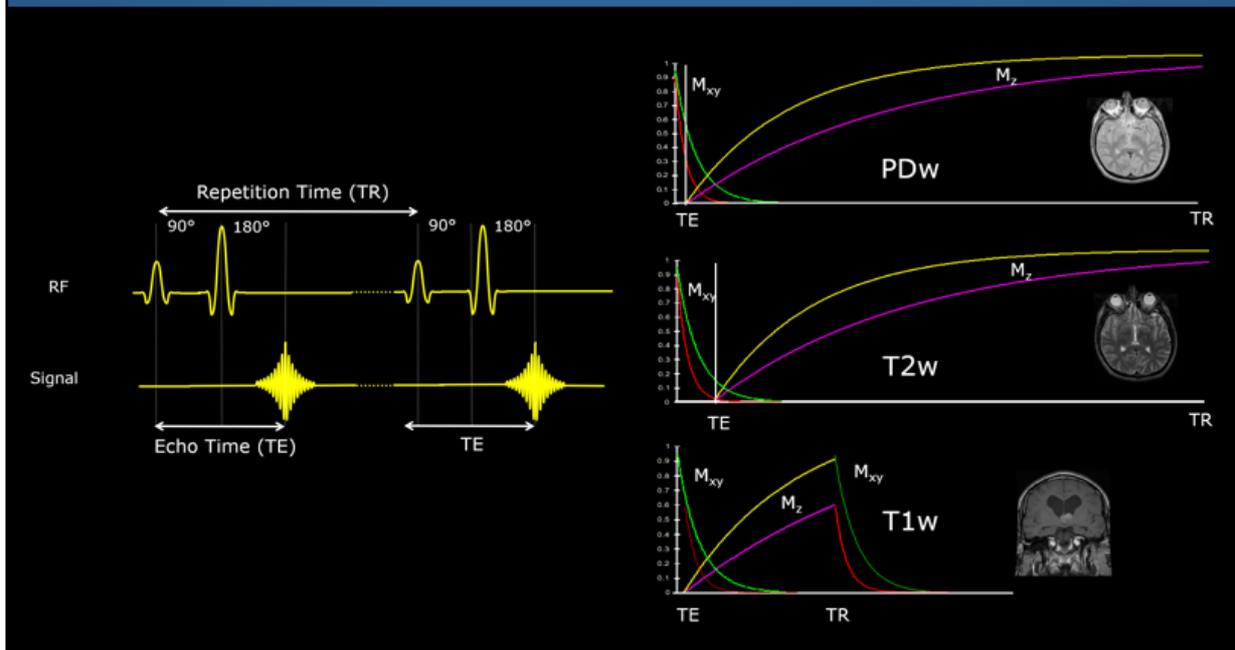
T₁ Relaxation Mechanism

As molecules tumble/vibrate the protons are exposed to an alternating magnetic field that stimulates relaxation. Relaxation is dependent upon the tumbling rate.



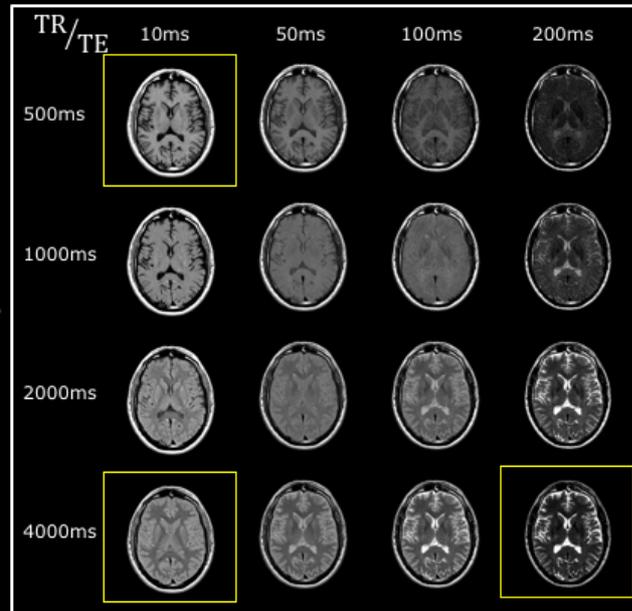
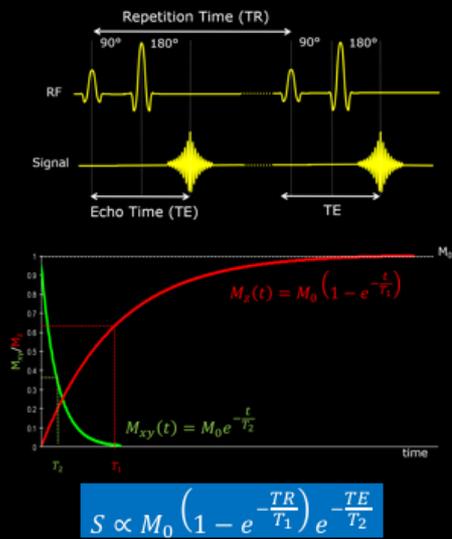
The alternating field comes from neighbouring protons or other nuclei or molecules, which have magnetic moments. In water the nearest adjacent nucleus will be the other hydrogen atom on the same molecule. Therefore, relaxation will primarily arise through the magnetic moment that one hydrogen nucleus 'sees' as it tumbles relative to the moment of the other hydrogen nucleus. This is often called an intra-molecular dipole-dipole interaction.

Spin Echo Contrast



The contrast in a spin echo sequence is controlled by the TE and the TR. The TE controls the amount of T_2 dephasing, i.e. T2-weighting in the image, whilst the TR controls the amount of T_1 recovery, i.e. T1-weighting in the image. We describe images as “weighted” since it is extremely difficult to have either zero TE to eliminate T_2 or very long TR, .e.g. at least 5 times the longest T_1 , to eliminate T_1 . In any case the underlying proton density differences are always present. A proton density (PD) weighted image therefore has a long TR to reduce T_1 effects and a short TE to minimise T_2 effects. A T2-weighted images has a long TR to minimise T1 effects but a long TE to maximise T_2 contrast. A T1-weighted images has a short TR to maximise the differences in T_1 relaxation but a short TE to minimise T_2 effects. Of course it is not possible to directly observe the longitudinal recovery so the magnetisation has to be tipped into the transverse plane in order to detect the T_1 differences. The inevitable T_2 decay during the time required even for a minimum value of TE, due to the time required to play out the RF pulses, results in reduced T_1 contrast.

Spin Echo Contrast



This slide shows the changes in contrast as TE and TR are varied. The main contrast weightings used clinically are T1w (top left hand corner), proton density (PDw) (bottom left hand corner) and T2w (bottom right hand corner)

Nuclear Magnetic Resonance Imaging



2003 Nobel Prize in Physiology or
Medicine for "for their discoveries
concerning magnetic resonance
imaging"

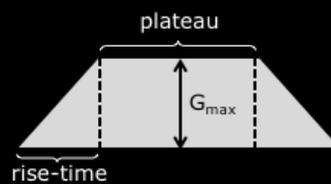
How do we go from NMR to MRI in order to create images. We firstly we have to drop this pesky nuclear bit. With the obvious association of nuclear with radioactivity, of which we have seen there isn't any in relation to NMR, nuclear got dropped early on in the development of imaging to just leave Magnetic Resonance Imaging or MRI. Two of the leading lights in the field were Professor Paul Lauterbur from the State University of New York and Professor Sir Peter Mansfield from the University of Nottingham who were subsequently awarded the 2003 Nobel Prize in Physiology or Medicine for "for their discoveries concerning magnetic resonance imaging"

Imaging & Gradients

- The key to MRI imaging is the concept of a magnetic field gradient

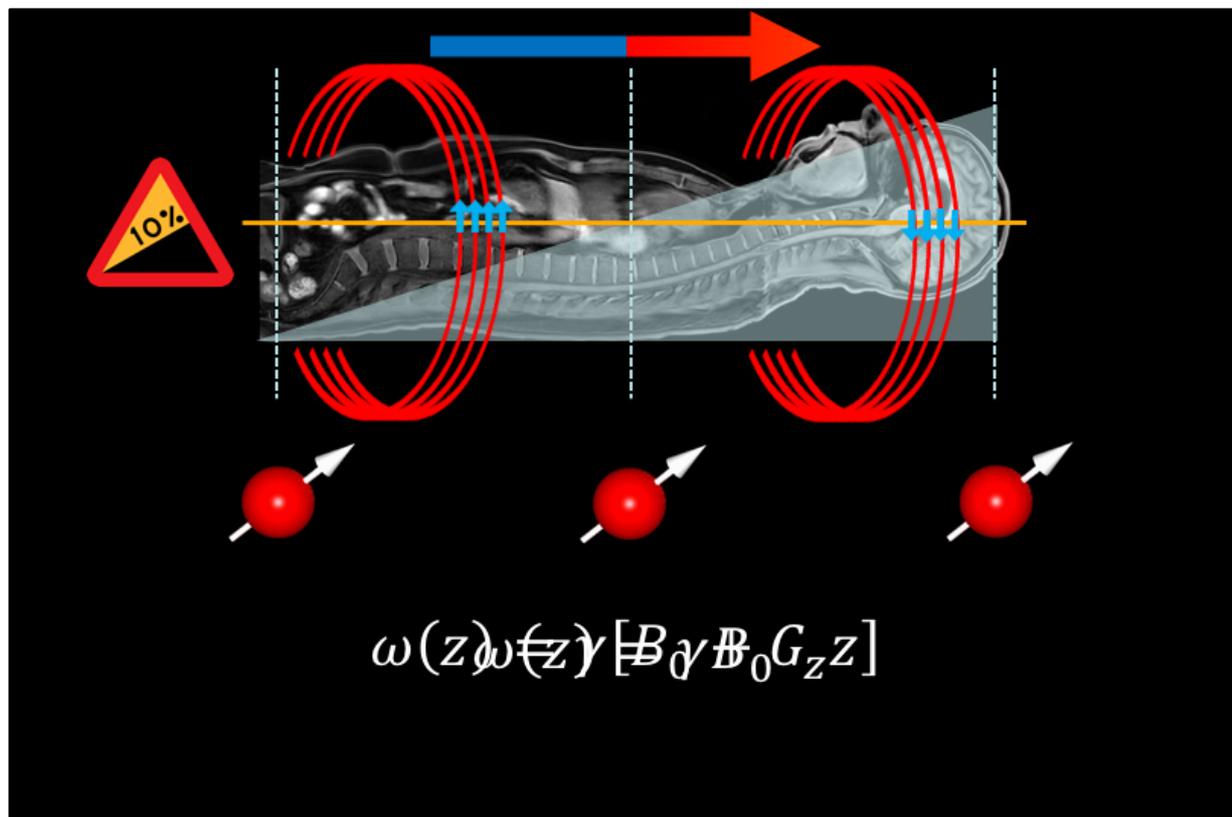
$G_x = \frac{\partial B_z}{\partial x}$, $G_y = \frac{\partial B_z}{\partial y}$, $G_z = \frac{\partial B_z}{\partial z}$ superimposed on the main static magnetic field

- The gradient creates a spatially dependent variation of the Larmor frequency, e.g. in the x-direction $B(x) = B_0 + G_x x$
- Gradient amplitudes are typically measured in mT/m or G/cm (10mT/m = 1G/cm)

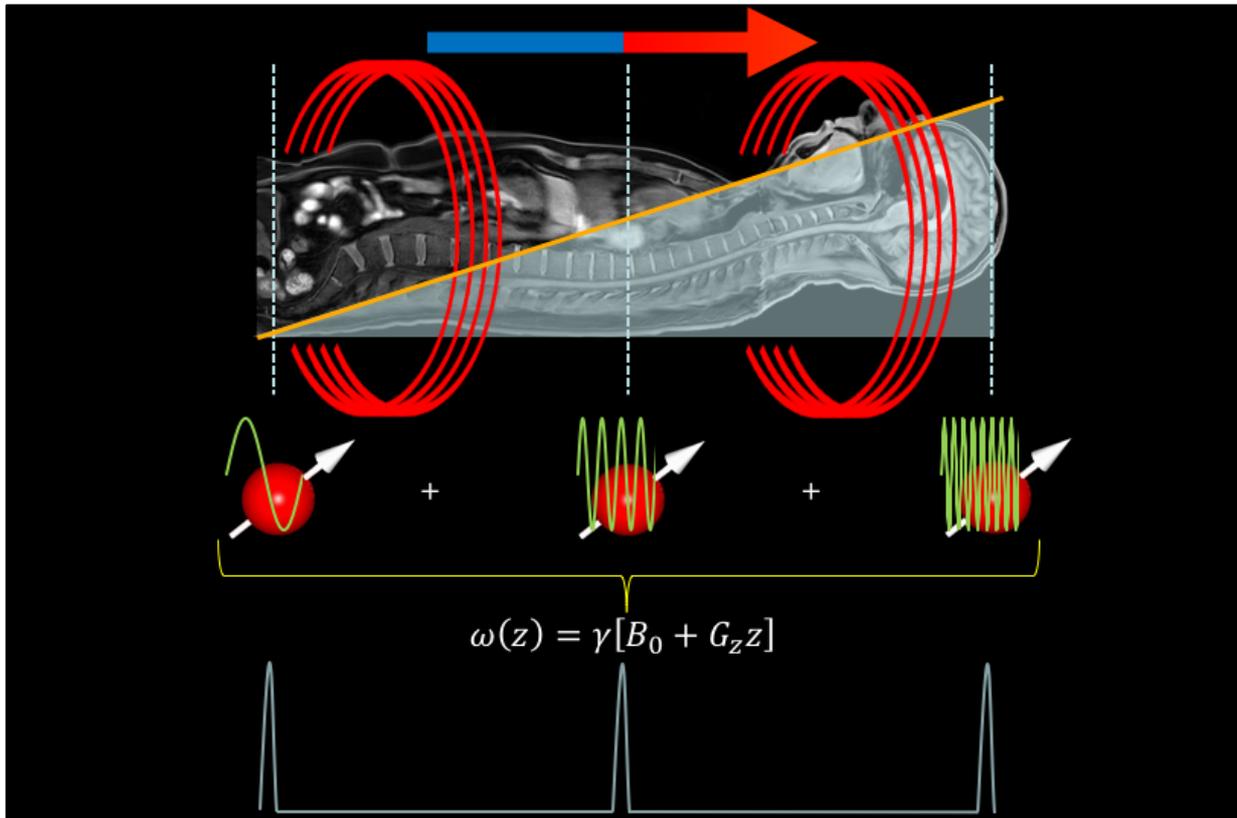


Paul Lauterbur
(1929-2007)

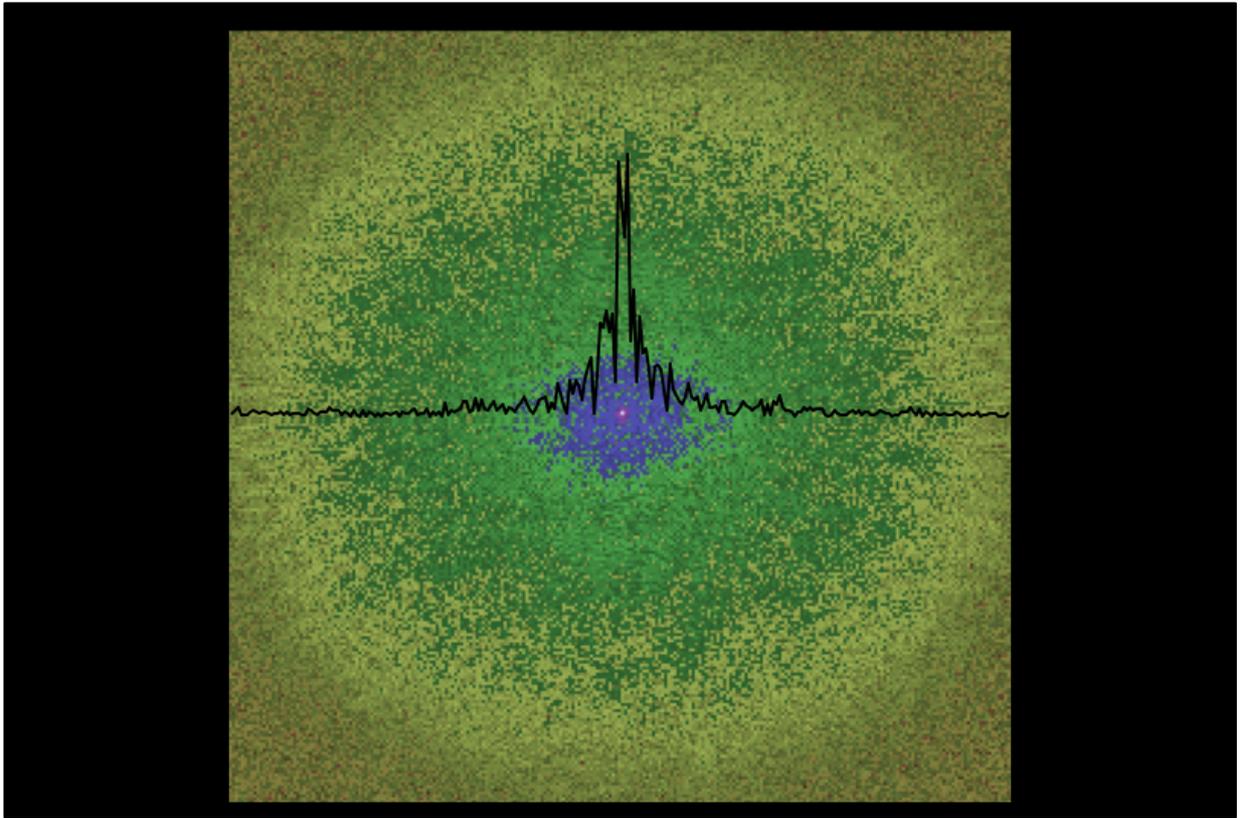
The crucial invention as far as imaging is concerned was made by Lauterbur who apparently had the idea whilst eating a hamburger. Lauterbur's great idea was to introduce a magnetic field gradient. He called his technique



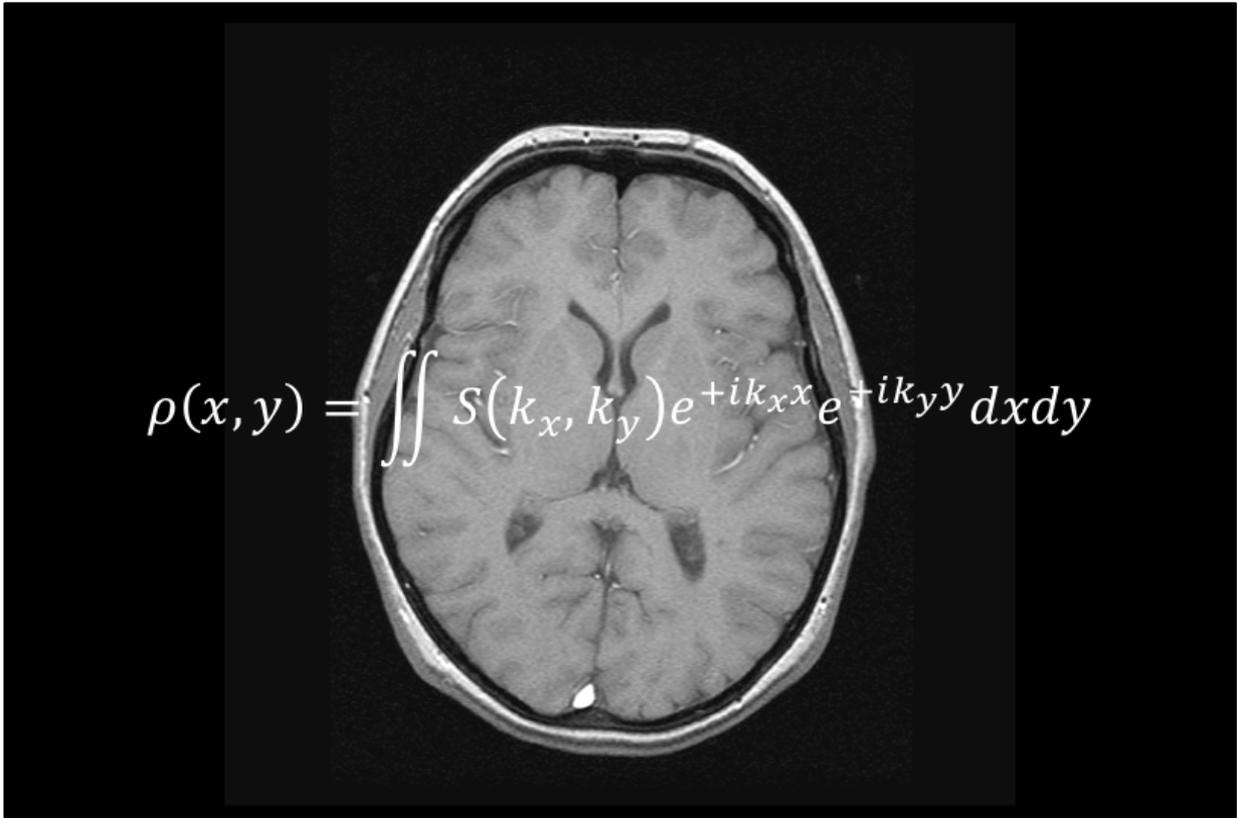
Here is our subject lying inside the magnet with the main magnetic field pointing from their feet to their head. At these three positions the main magnetic field is constant and the protons are all precessing at the same frequency. If we add yet more coils inside the MRI scanner then by passing current through these coils we can create a gradient, that is to say a slope in the magnetic field in addition to the main magnetic field. It is the switching on and off of this gradient that causes the characteristic knocking noise when imaging. These gradient coils physically flex, not by a lot, but enough to create the noise. You can see that when the gradient is applied there is now a difference in the precessional frequencies because the protons are now in slightly different magnetic fields due to the gradient.



It may be easier to understand in terms of musical notes. The proton on the left precesses at the lowest frequency, the proton in the middle at a slightly higher frequency whilst the proton on the right precesses at the highest frequency. Now the signal comes from all the protons so what we detect in this case is the sum of all three frequencies. How can we separate the individual notes from the sum? A Fourier transform can transform a time domain signal to the frequency domain. So we can see three peaks. The area under each peak would correspond to the weighted NMR signal at that position. If the gradient is accurately calibrated then we can go from frequency directly to spatial position

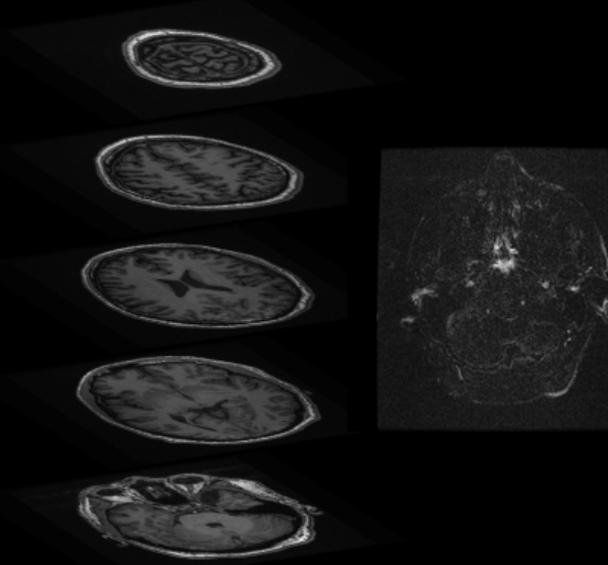
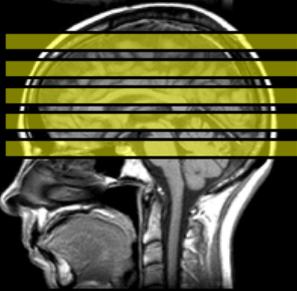
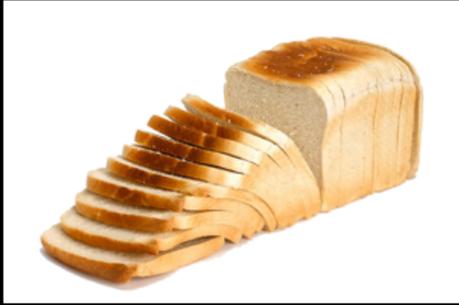


The real signal from a human subject looks like this and can be displayed as one line of a raw data image it looks like this. Now the complete MRI scan involves changing these gradients many hundreds of times and detecting the signals across the entire body. So the scanner acquires data like this. This is the raw data, sometimes called *k*-space.



Simply performing an inverse 2D Fourier transform of this raw data reconstructs the image.

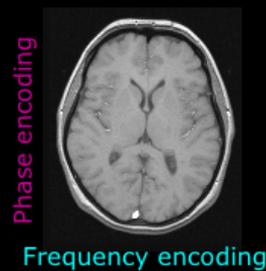
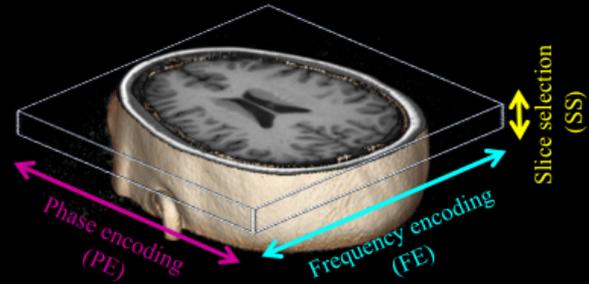
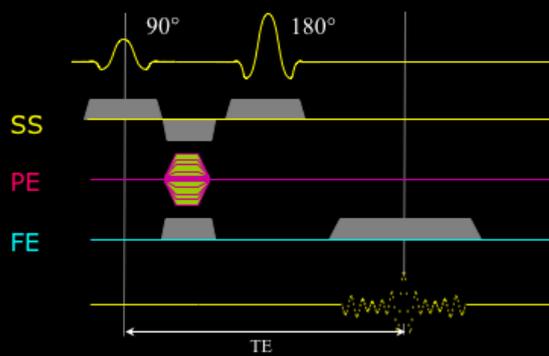
Slices



Images are usually acquired and displayed as “slices” or “sections” through the data volume. A brain MRI for examples would involve the acquisition of a number, typically 20-30 of relatively thin, e.g. 5mm slices throughout the brain.

Spatial Localisation

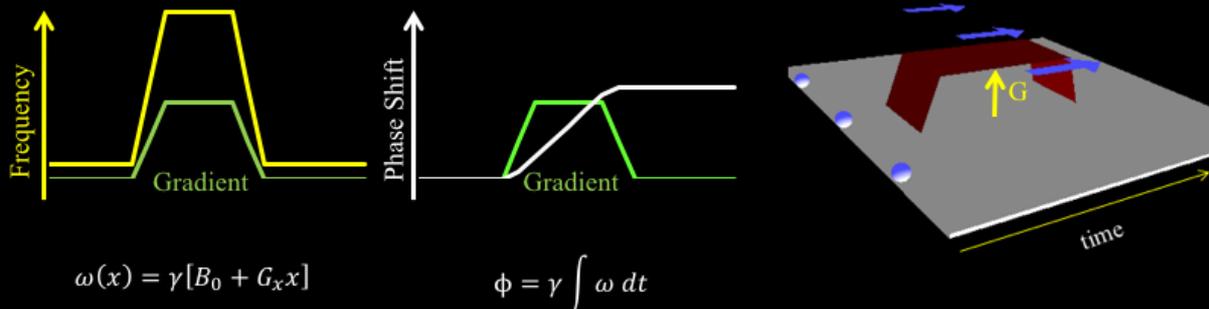
Spatial encoding of the MR signal is achieved through the use of magnetic field gradients



The acquisition of an MRI image involves the combination of RF pulses to tip the net magnetisation, various time delays to allow the desired contrast, e.g. T1w, to develop and gradient pulses to spatially encode the MR signal. The gradients are spatially-linear magnetic fields that are superimposed upon the static magnetic field. There are three orthogonal physical gradients in x, y and z but we will refer to them by the logical names of slice selection, phase encoding and frequency encoding. In this spin echo pulse sequence we can see these gradients represented as trapezoidal blocks. The trapezoidal shape means that it takes a finite time (ramp up) to switch on the gradient, it is then held on (the flat top), for a period of time before it ramps down.

Gradient Effects

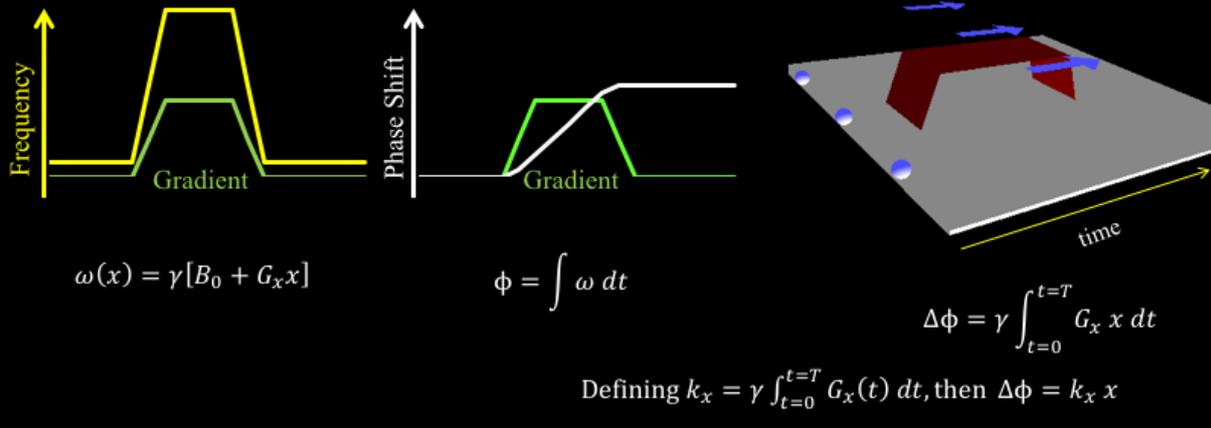
Gradient pulses induce a change in Larmor frequency with amplitude/position, and phase accumulation with time



A magnetic field gradient will not only cause a transient change in Larmor frequency but will also induce a (relative) phase shift in the transverse magnetisation proportional to the strength and duration, e.g. "area", of the gradient pulse. The diagram shows three spins initially all precessing in phase (we will ignore spin dephasing/relaxation etc). The middle spin will then come under the influence of a magnetic field gradient pulse (in this case we assume the x-direction), whereas the others will not. Note how the spin precesses more rapidly due to the increased Larmor frequency. However at the end of the gradient pulse all three spins precess at the same frequency but the middle spin is 180° out of phase compared to the other two.

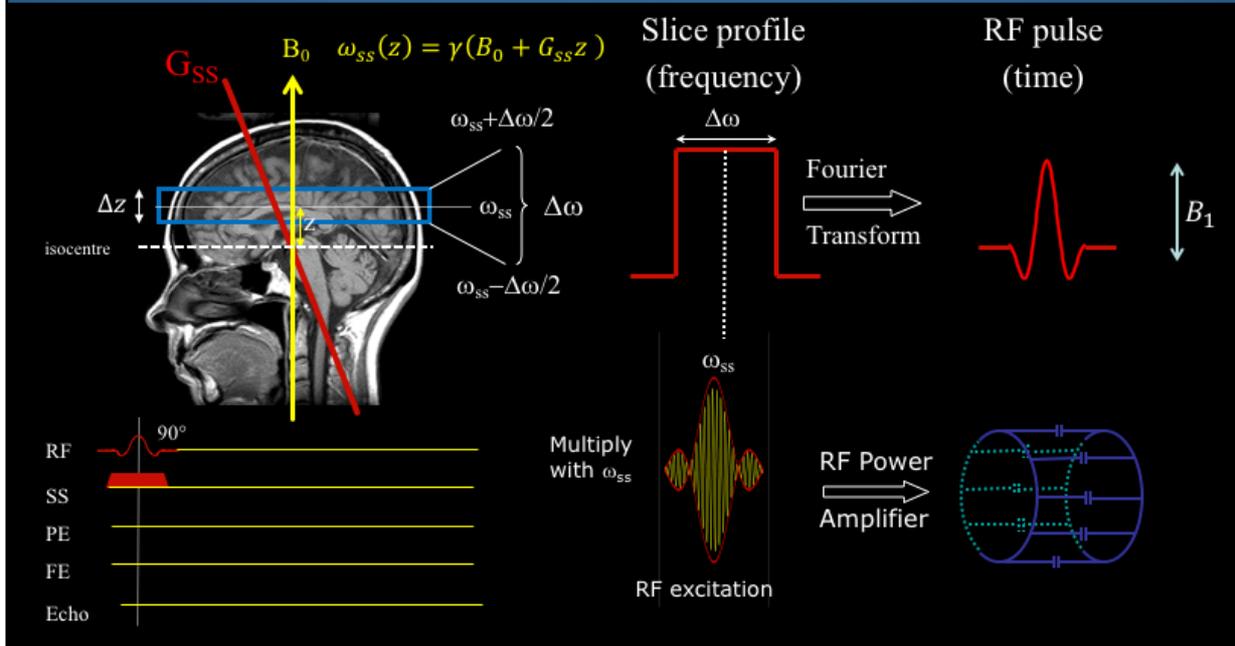
Gradient Effects

Gradient pulses induce a change in Larmor frequency with amplitude/position, and phase accumulation with time



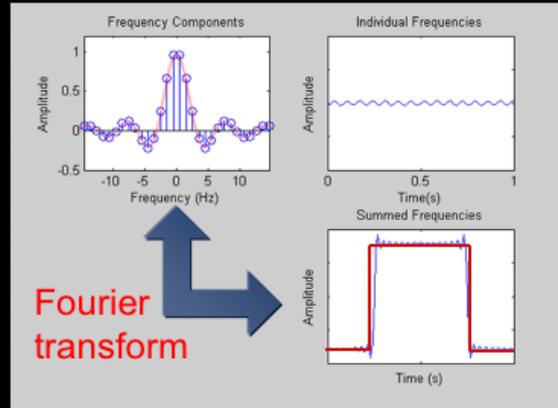
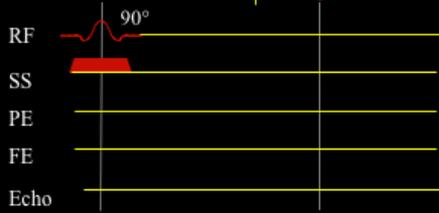
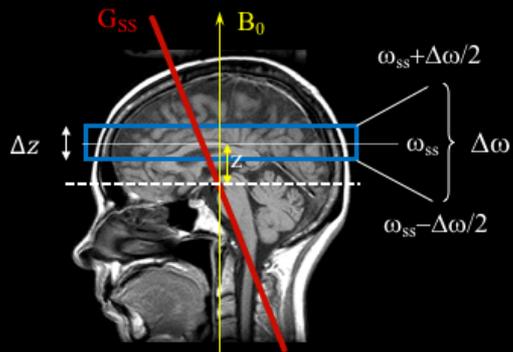
This phase shift can be represented in terms of a k -space with units of inverse distance

Slice Selection

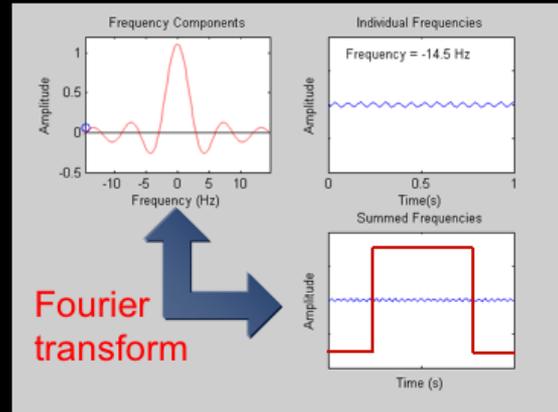
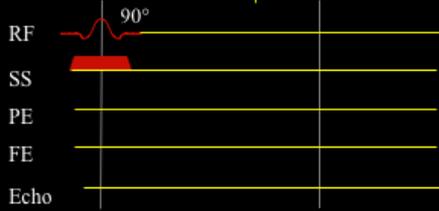
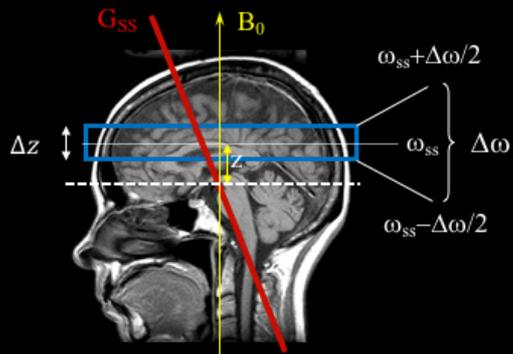


A slice through the body is selected by the application of an RF pulse together with a magnetic field gradient in the slice select direction. In this case an axial slice through the brain. We wish to only excite those spins at a particular physical offset (z) from the centre of the magnet known as the isocentre, i.e. the fulcrum of the gradients. We wish the slice to have a certain thickness (Δz), typically 5-10mm. From the Larmor equation we know that the spins within the slice will be precessing at $\omega_{ss} \pm \frac{\Delta\omega}{2}$, where $\omega_{ss}(z) = \frac{\gamma}{2\pi}(B_0 + G_{SS}z)$. The slice profile in the frequency domain will be a top hat function, which will be a sinc function ($\text{sinc}(x) = \frac{\sin(x)}{x}$) in the time-domain. The sinc envelope is used to amplitude modulate the transmit frequency ω_{ss} . If the amplitude and duration of the RF pulse are sufficient to nutate the magnetisation by 90° then all the spins that are resonant at $\omega_{ss} \pm \frac{\Delta\omega}{2}$ will be tipped into the transverse plane.

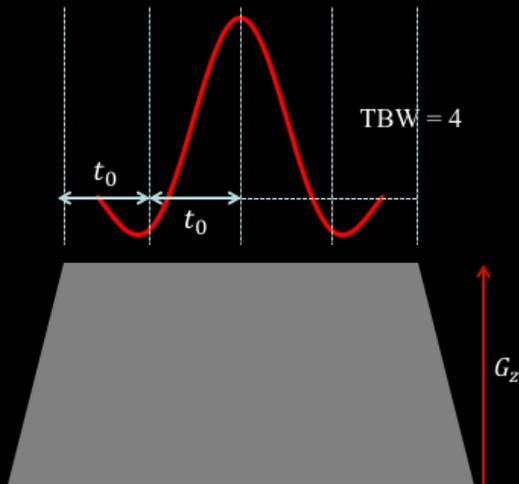
Slice Selection



Slice Selection



Slice Select Gradient Amplitude



To a good approximation
the bandwidth is given by

$$\Delta f \approx \frac{1}{t_0}$$

Time-Bandwidth Product (TBW)

= no. of zero crossings

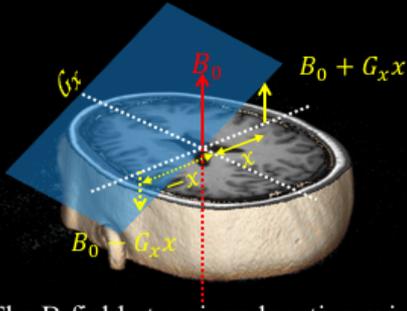
$$T\Delta f = N_L + N_R$$

$$\Delta f = \frac{T}{N_L + N_R}$$

$$G_z = \frac{\Delta f}{\gamma \Delta z}$$

A sinc pulse with a TBW=4 and duration of $T=3.2\text{ms}$ has four zero crossings with $t_0=0.8\text{ms}$. This results in a bandwidth of $\Delta f=1250\text{Hz}$, i.e. all spins within a bandwidth of $\omega_{SS} \pm \frac{1250}{2}\text{Hz}$ will be tipped. A slice thickness of 5mm therefore requires a z-gradient amplitude of $G_z = 5.87\text{mT/m}$. Usually the RF bandwidth is fixed and the gradient amplitude is set by the operator defining the slice thickness. A very thin 2D slice prescription may be limited by the maximum gradient amplitude achievable. A thin slice will also have a relatively poor signal-to-noise ratio (SNR)

In-plane Spatial Localisation



- The B field at a given location x is:
- $B(x) = B_0 + G_x x$
- $\omega(x) = \gamma[B_0 + G_x x]$
- In a reference frame precessing at ω_0
- $\omega(x) = \gamma G_x x$

- The solution of the Bloch equation in complex notation (ignoring T_2) is
- $M(x, t) = M_0(x)e^{-i\omega(x)t} = M_0(x)e^{-i\gamma G_x x t}$
- The spin density at location x is $\rho(x)$, in which case the signal S between x and dx is given by

$$S(x) = \int_x^{x+dx} \rho(x) e^{-i\gamma G_x x t} dx$$

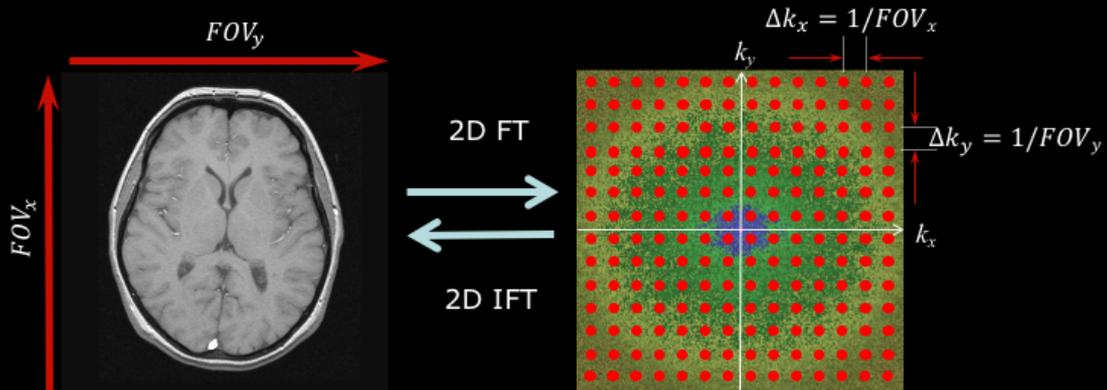
i.e., the signal is the Fourier transform of the spin density

- For a continuous 2D distribution $\rho(x, y)$
- $S(x, y) = \iint \rho(x, y) e^{-i\gamma G_x x t} e^{-i\gamma G_y y t} dx dy$

If we initially consider a gradient in the x-direction (left-right across the subjects head) then the gradient causes a spatially dependent change in the Larmor frequency which means that the MRI signal is the Fourier transform of the objects transverse magnetization.

k -space

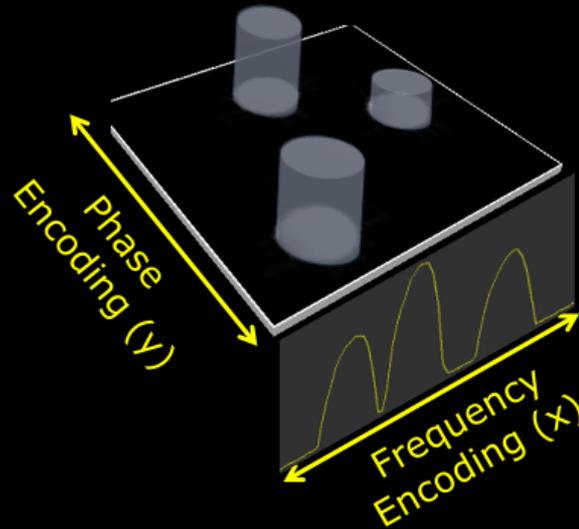
- The spatial frequencies k_x and k_y can be defined as
 - $k_x = \gamma \int G_x(t) dt$
 - $k_y = \gamma \int G_y(t) dt$
- For simple constant amplitude gradients
 - $k_x = \gamma G_x t$ and $k_y = \gamma G_y t$
 - $S(k_x, k_y) = \iint \rho(x, y) e^{-ik_x x} e^{-ik_y y} dx dy$



It is usual in MRI to define the k -space trajectory in terms of the areas of the gradient waveforms. This provides the basis of our 2D discrete sampling pattern. The raw data array shown on the right is often referred to as k -space. MR image reconstruction is a simple 2D inverse Fourier transformation. The reconstruction is generally performed using a fast Fourier transform, hence why image acquisition matrices are usually in powers of 2, e.g. 128, 256 or 512. It should be noted that MRI raw data is actually complex. The raw data consists of multiple “lines” (k_y) of echo (k_x) data.

I am assuming that $\gamma = \frac{\gamma}{2\pi}$

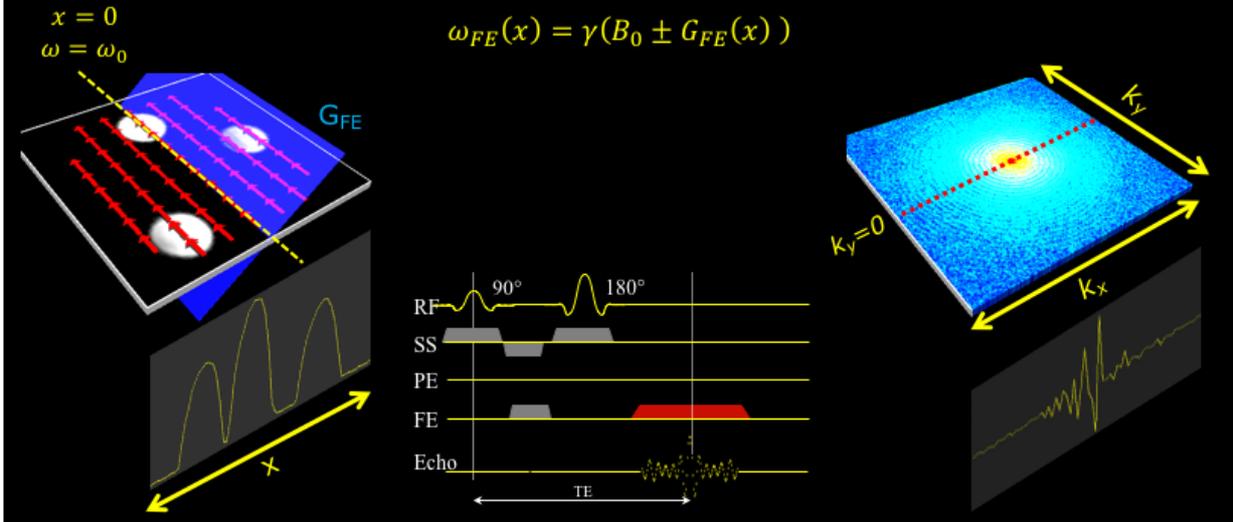
1D Projection



To explain the principles of MR imaging let us consider something slightly easier than a brain - three vials of water. Since the vials contain different volumes of water they contain are different proton densities represented by the different heights of the three peaks that represent a 1-dimensional projection through the data in the frequency encoding direction. We have no information as to their relative positions in the orthogonal phase-encoding direction.

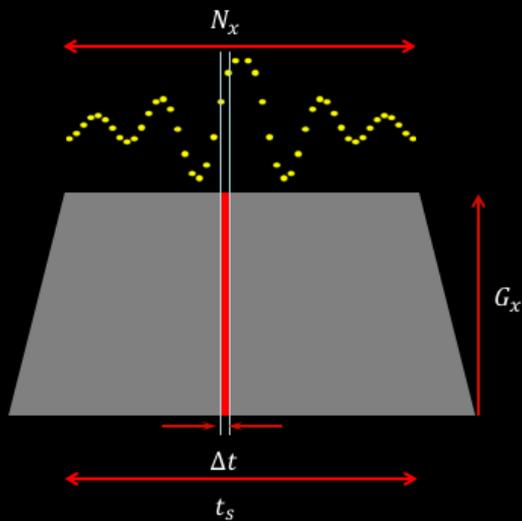
Frequency Encoding (FE)

In a frame of reference rotating at ω_0



In the rotating frame of reference and ignoring any signal changes due to relaxation etc., the spins that have been tipped into the transverse plane by the RF pulse will all precess at ω_0 . However if we apply the frequency encoding (FE) gradient, in this case along the x -axis, the spins will precess at different rates depending upon their position along the x -direction. Since we are in the rotating frame of reference at ω_0 the spins in the centre appear stationary whilst the spins on either side are precessing at different rates and in opposite senses. The spin echo that forms in the presence of this frequency encoding gradient represents a set of spatial (k_x) frequencies. This echo is the Fourier transform of the 1D projection. In our 2D k -space this single "line" of echo data will go through the centre. This is often known as the central slice theorem.

Frequency Encoding Gradient Amplitude



$$\Delta k_x = \gamma G_x \Delta t$$

$$FOV_x = \frac{1}{\Delta k_x}$$

$$t_s = N_x \Delta t$$

$$\Delta k_x = \gamma G_x \frac{t_s}{N_x}$$

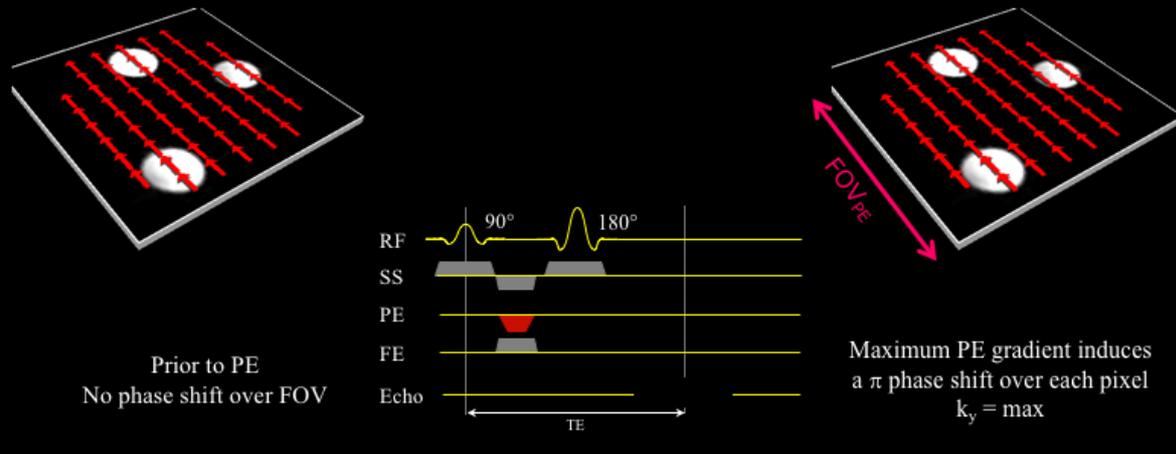
$$\text{Receiver bandwidth (RBW)} = \frac{1}{\Delta t} = \frac{N_x}{t_s}$$

$$G_x = \frac{\text{RBW}}{\gamma \text{FOV}_x}$$

A typical receiver bandwidth is $\pm 16\text{kHz}$ so $\text{RBW} = 32\text{kHz}$. If we acquire $N_x = 256$ samples then $\Delta t = 0.03125\text{ms}$ and the total data acquisition time $t_s = 8\text{ms}$. For a 24cm field-of-view (FOV) the frequency encoding gradient amplitude required is $G_x = 3.13\text{mT/m}$. Very small fields-of-view may be limited by the available maximum gradient amplitude, assuming you have the available SNR.

Phase Encoding (PE)

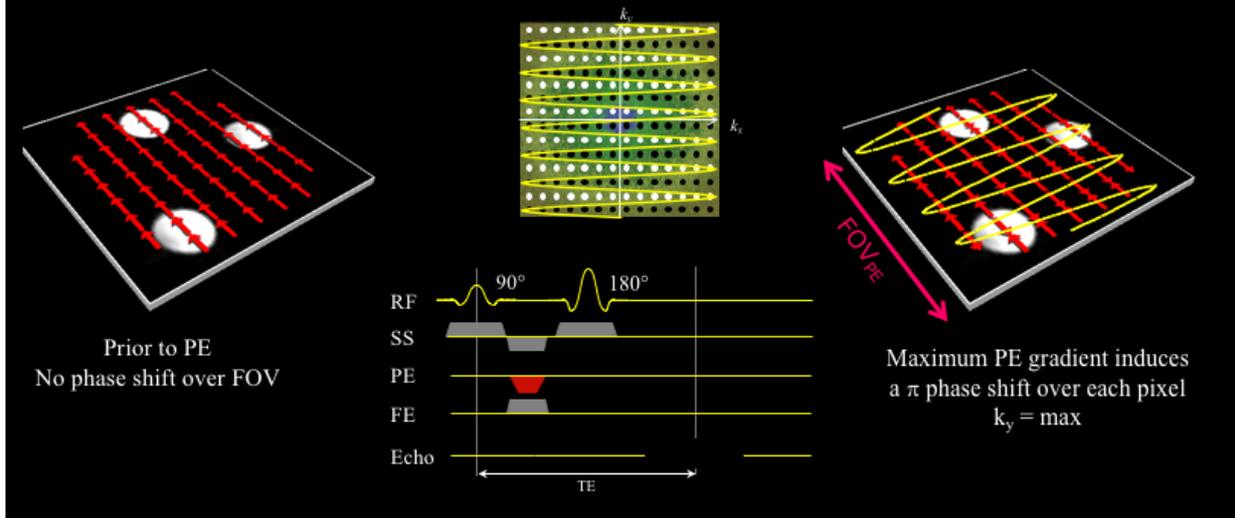
A phase encoding gradient is used to induce a phase shift across the field-of-view in the y-direction to create a pseudo-spatial frequency (k_y)



In order to encode other “lines” of k -space we induce “pseudo-frequencies” in the phase encoding or y -direction. The phase encoding (PE) gradient is applied prior to the frequency encoding gradient and introduces a phase “twist” into the spins along the y -direction. During imaging the spin echo pulse sequence is repeated k_y times with the phase encoding gradient being stepped through a number of values, from maximum negative to maximum positive. In this example the amplitude and duration of the PE gradient, i.e. the area, is such that a π phase shift is induced over each pixel. This is the maximum phase encoding step, i.e. $k_y = \max$

Maximum Phase Encoding

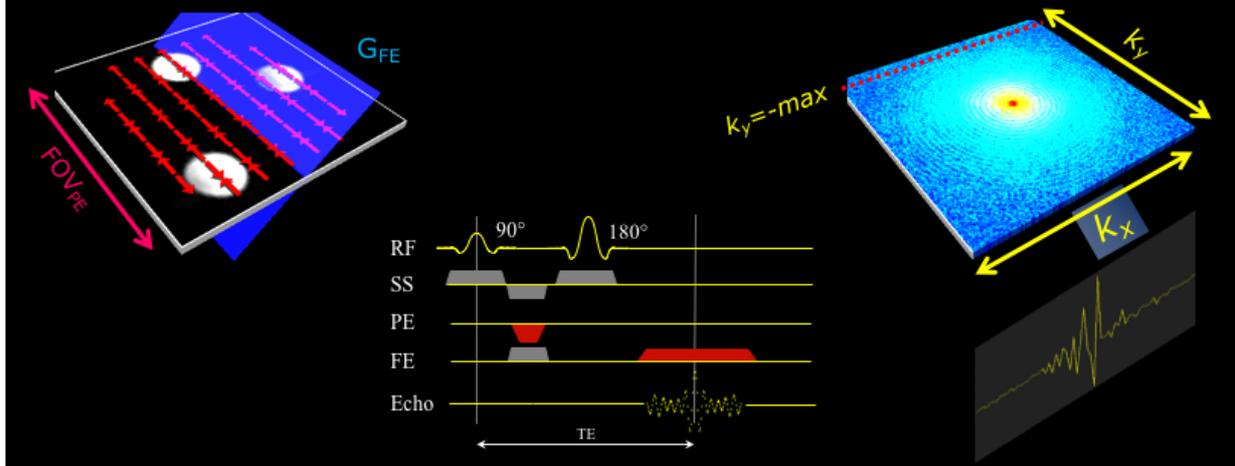
The maximum PE gradient induces a π phase shift over each pixel



We can see that this maximum negative PE gradient has caused a π phase shift to be induced over each pixel. This represents the highest possible spatial frequency and hence the spatial resolution of the image in the y-direction.

Maximum Phase & Frequency Encoding

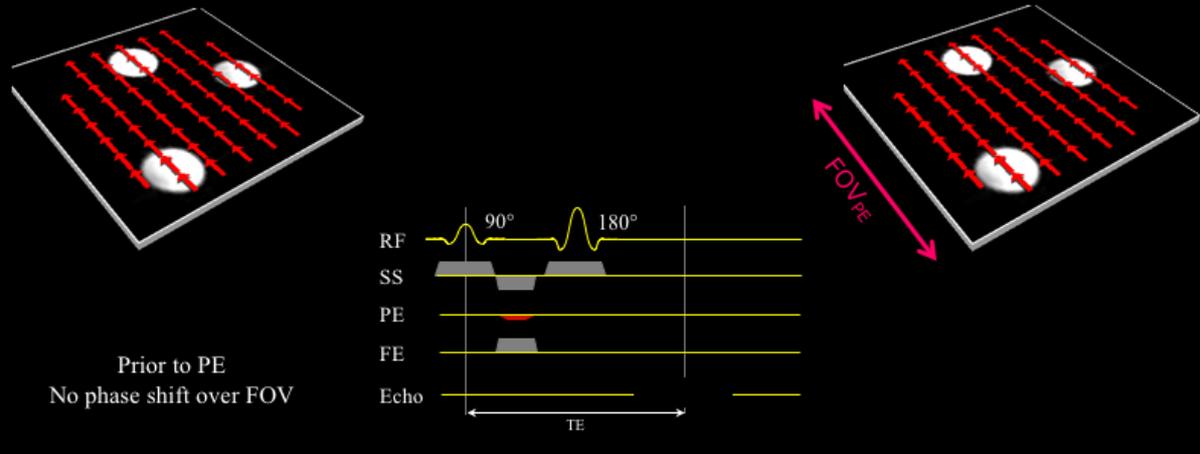
The phase encoded spins are then frequency encoded
($k_y = \text{max}$)



The y-direction phase shift induced by the PE gradient is retained during the formation of the echo in the presence of the FE gradient. In our 2D k-space this "line" of echo data will be assigned to the top of the raw data matrix, since it is acquired with maximum phase encoding, i.e. $k_y = -\text{max}$.

Minimum Phase Encoding

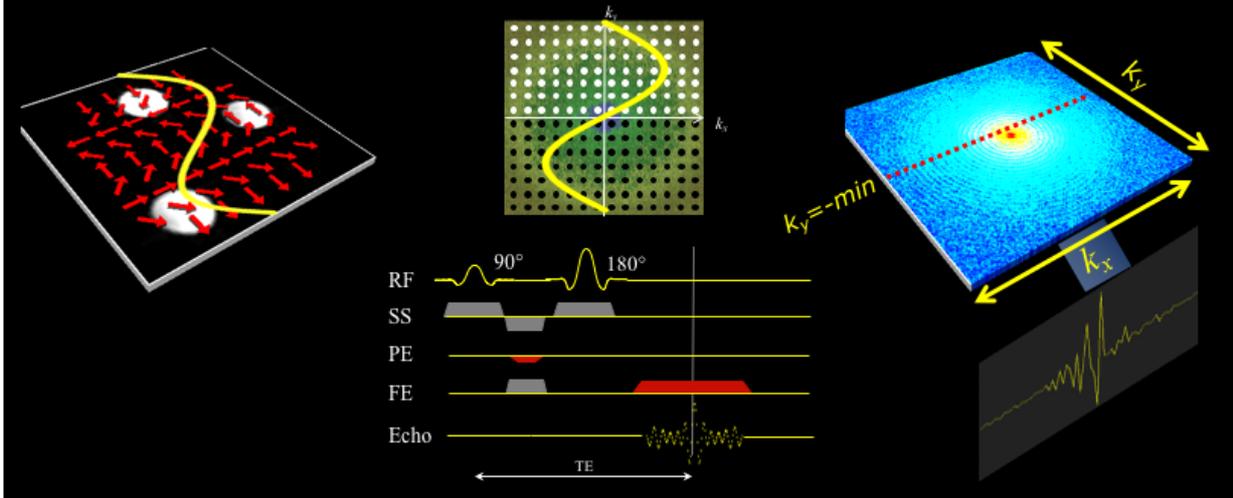
The minimum PE gradient induces a 2π phase shift over the FOV
($k_y = \min$)



The phase encoding gradient is stepped k_y times equally, from the maximum negative value to the maximum positive value. In this example we consider the minimum negative value. We can see that this maximum negative PE gradient has caused a

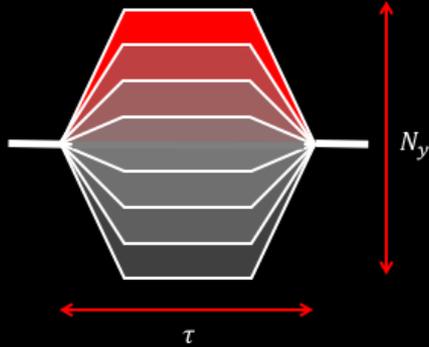
Minimum Phase & Frequency Encoding

The minimum PE gradient induces a 2π phase shift over the FOV
($k_y = \text{min}$)



We can see that this minimum negative PE gradient has caused a 2π phase shift to be induced over each pixel. Following frequency encoding in our 2D k-space this "line" of echo data will be assigned nearest the centre of the raw data matrix, since it is acquired with minimum phase encoding, i.e. $k_y = -\text{min}$.

Phase Encode Gradient Area



$$k_{y,max} = \gamma \int_0^{\tau} G_{y,max} dt$$

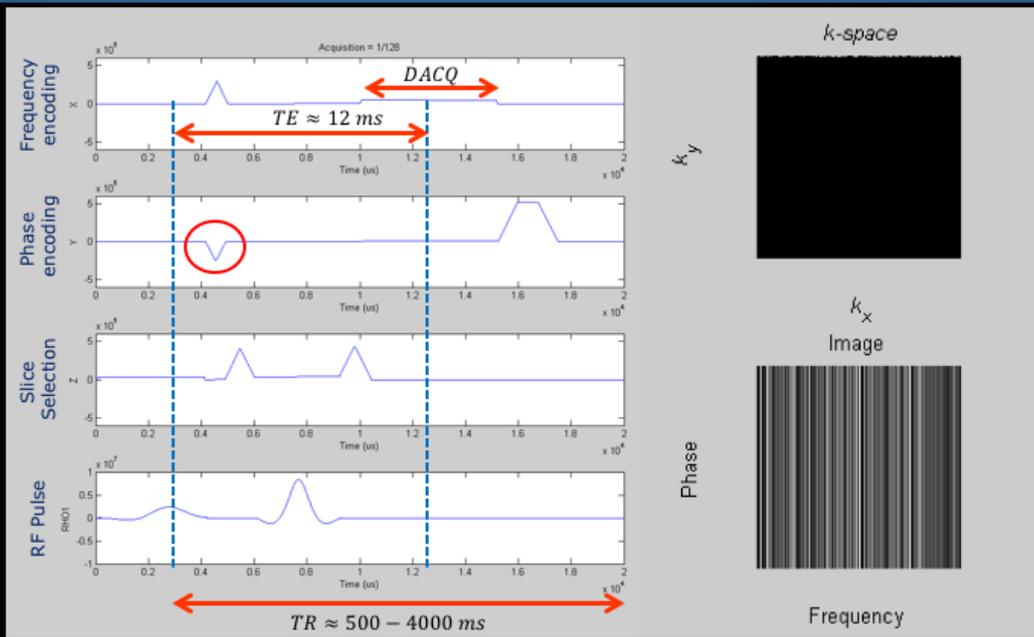
$$k_{y,max} = \frac{1}{2} (N - 1) \Delta k_y$$

$$FOV_y = \frac{1}{\Delta k_y}$$

$$A_{y,max} = \frac{(N_y - 1)}{\gamma FOV_y}$$

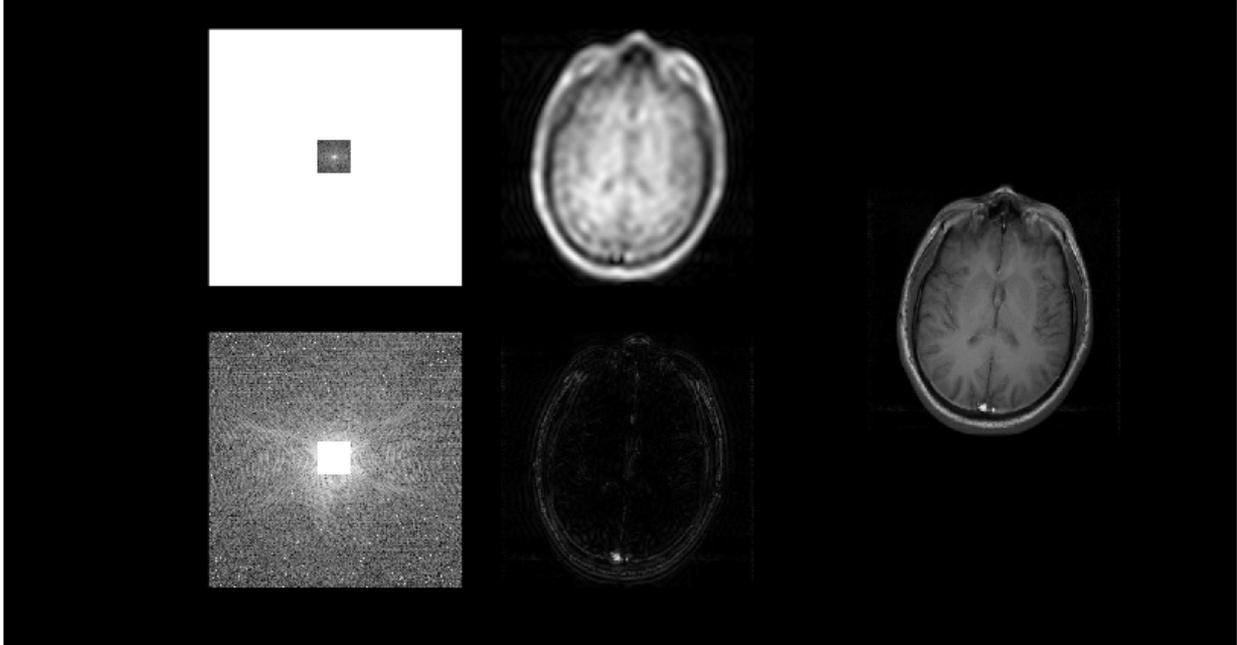
The phase encode gradient needs to be stepped through N_y steps. For each phase encoding the data is subsequently frequency encoded and the data placed in the corresponding part of k-space prior to 2D Fourier transformation

Spin Echo Pulse Sequence



This movie shows a spin echo sequence being repeated 128 times. Note that all the gradients are the same for each repetition except for the phase encoding gradient (red circle) that is stepped ± 64 times from maximum negative through to maximum positive. Each echo is placed in the appropriate position in k -space and in this example I have performed a Fourier transform of the partially filled k -space to show the developing appearance of the image. In standard MRI the image is not reconstructed until all of k -space is filled. This pulse sequence shows the first 20ms of a spin echo sequence. The actual TR, to allow for T_1 recovery and hence create a T1-weighted image, is 500ms. Therefore the actual acquisition time for this image is $500\text{ms} \times 128 = 64\text{s}$.

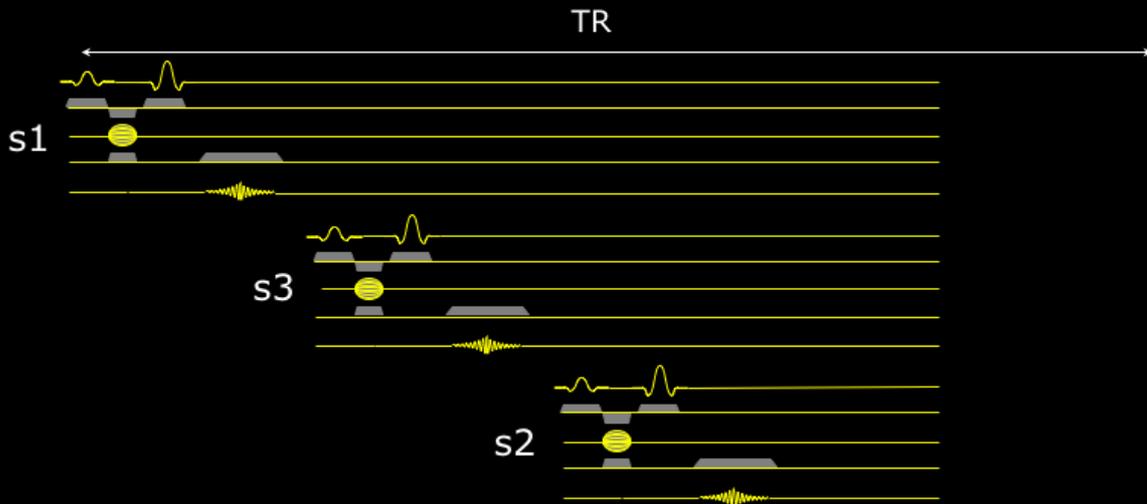
Spatial Frequencies



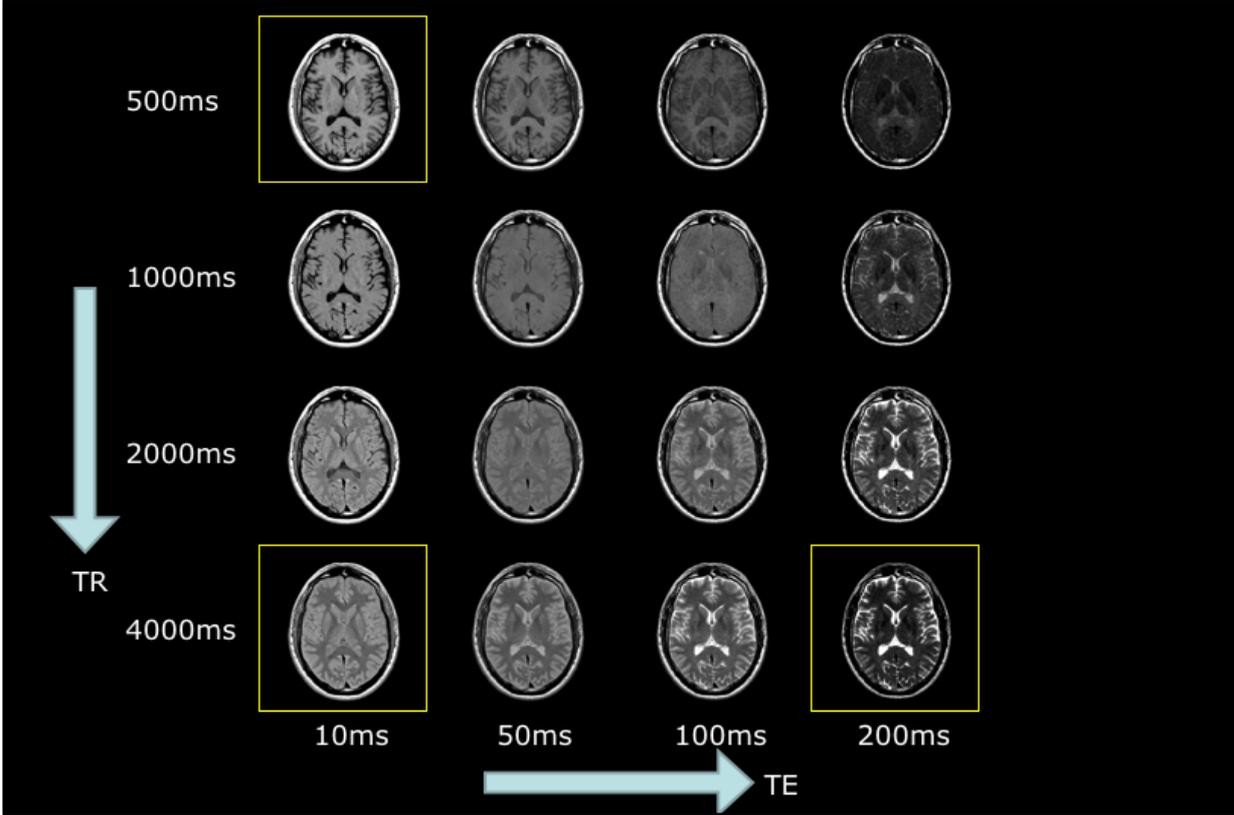
k -space is the spatial frequency representation of the image. If we just reconstruct data from the centre of k -space then you get a high SNR image but with very poor spatial resolution. Conversely reconstructing the periphery of k -space results in an image showing the high spatial frequencies, i.e. edges in the image but no bulk contrast. The full k -space gives a reasonable image.

Multi-slice Imaging

$$\text{Scan Time} = TR \cdot N_y \cdot NSA$$

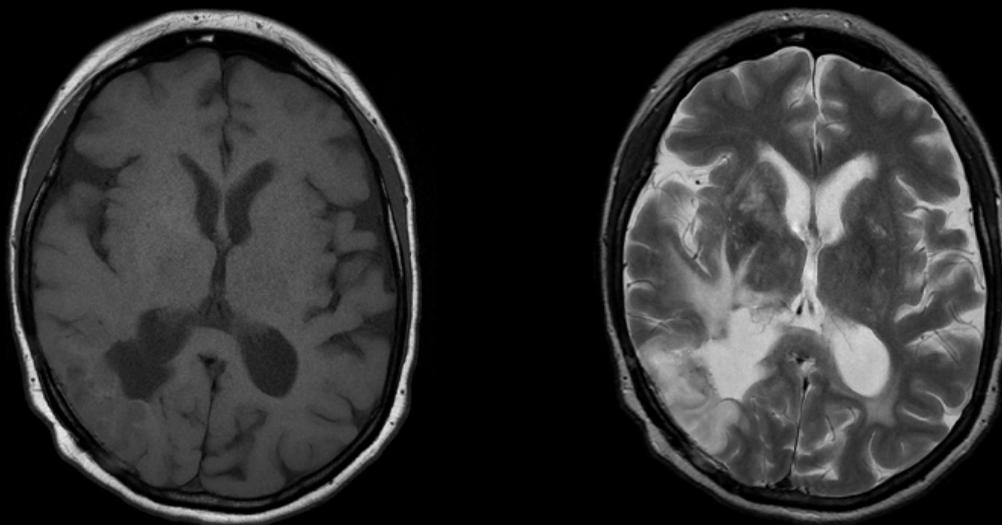


Since TR is usually much longer than T_1 there is substantial dead-time in a pulse sequence whilst we wait for T_1 recovery. The minimum acquisition time is given by $TR \cdot N_y \cdot NSA$ where NSA is the number of signal averages. Signal averaging is a commonly used method to improve the SNR in an image, since the signal is coherent and the noise should be incoherent the SNR will improve proportional to \sqrt{NSA} . During the recovery period other slices can be excited and acquired, simply by repeating the acquisition and changing the excitation frequency. Slices are usually acquired in odd then even order to avoid too much "cross-talk" between adjacent slices due to imperfections in the slice profile.

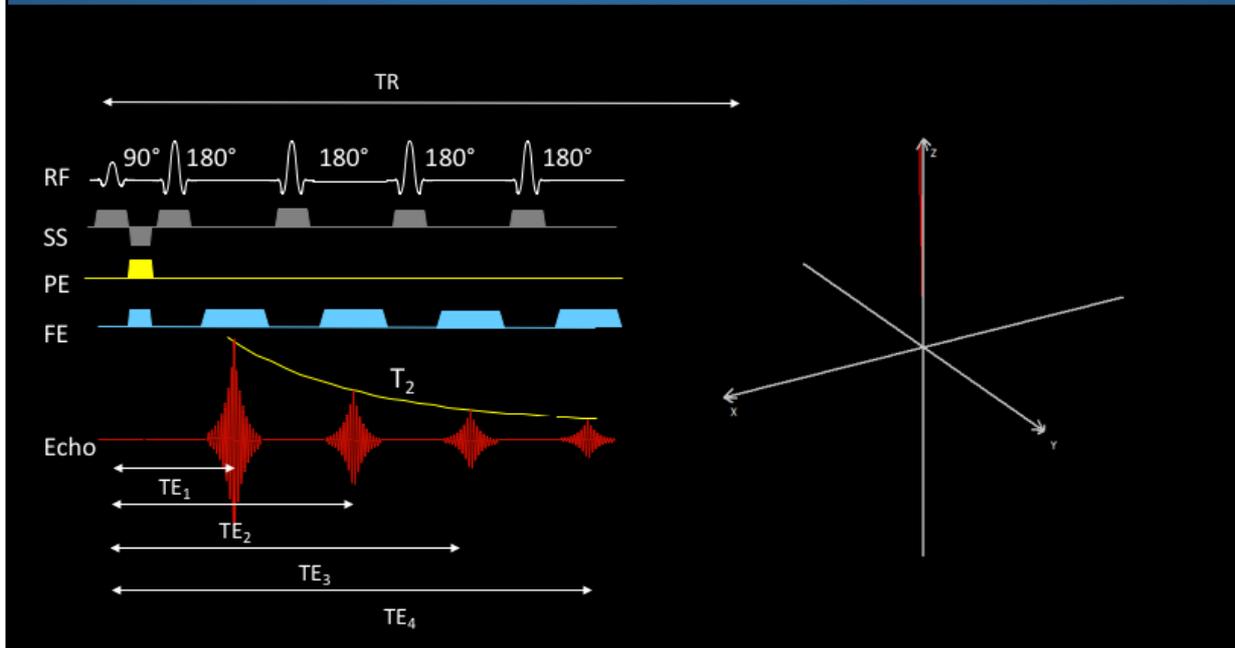


A refresher on the effects of TE and TR in a spin echo sequence.

T1w vs. T2w

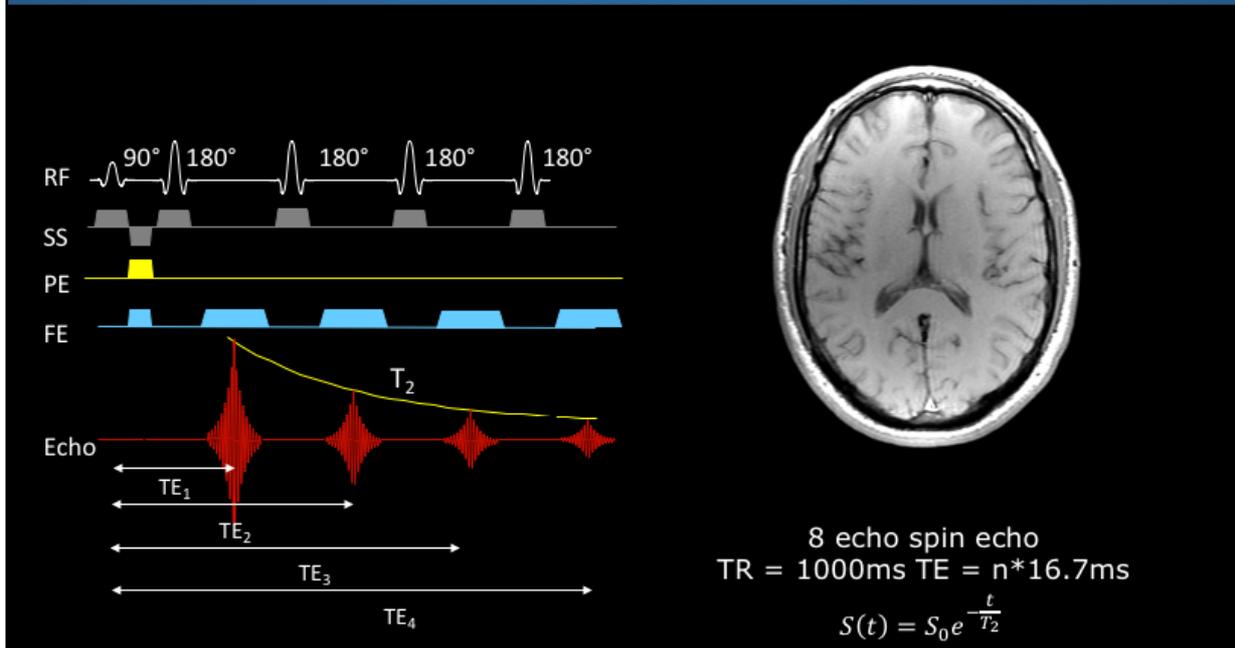


Multi-Echo Spin Echo



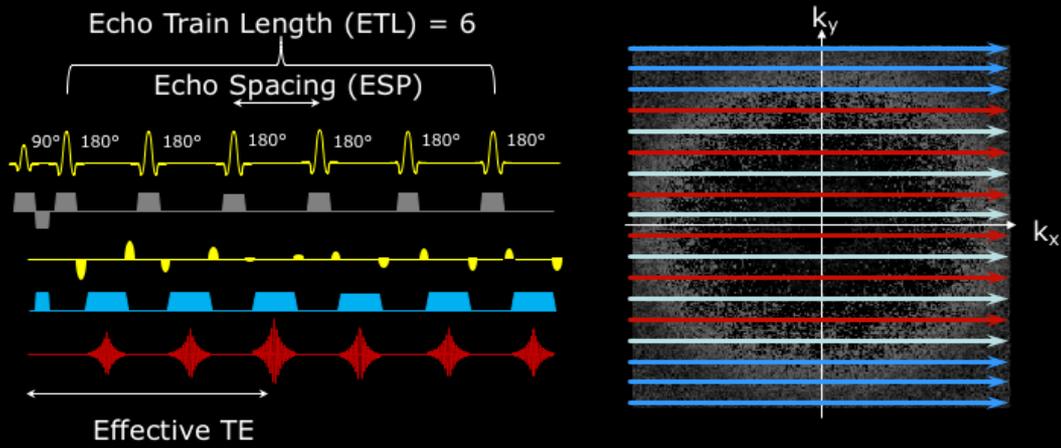
It is possible to apply multiple 180° refocusing pulses to acquire multiple spin echo images. The phase encoding is applied only once per TR but each echo is placed into its own separate k-space.

Multi-Echo Spin Echo



Each image is acquired at a slightly different TE. It would be possible to calculate a quantitative T2 image on a pixel by pixel basis by fitting the equation $S(t) = S_0 e^{-\frac{t}{T_2}}$ to the pixel data from each echo.

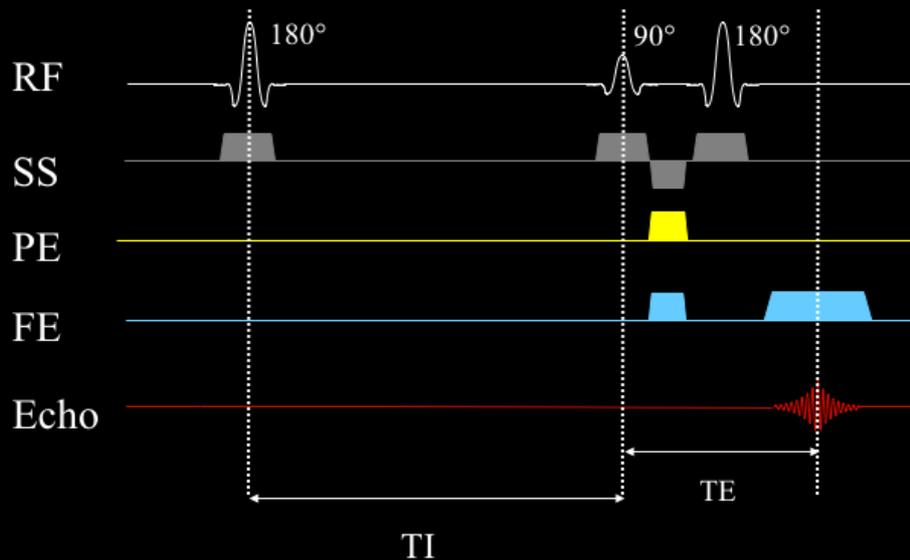
Fast Spin Echo (FSE)



$$\text{Scan Time} = \frac{TR \cdot N_y \cdot NSA}{ETL}$$

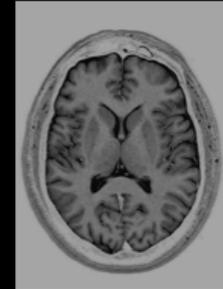
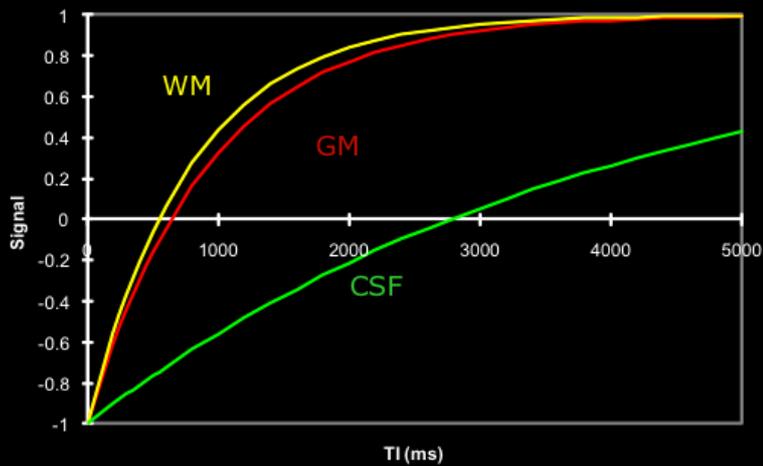
For very long repetition times, e.g. 5-10s then acquisition times can be very long. This problem has been addressed by using the concept of multiple spin echoes but individually phase encoding each echo and using it in the same k-space. This is the basis of fast (or turbo) spin echo. In this example six echoes are individually encoded. Note that each phase encoding is “unwound” by applying an equal but opposite polarity gradient pulse after each frequency encoding. This effectively resets the phases ready for the next encoding. The overall scan time is reduced by the number of echoes encoded per TR, known as the Echo train length (ETL). In this case the ETL is 6 but for long TR acquisitions ETLs from 12-32 may be used. Since multiple echoes are used in the same k-space what is the TE? As we know the bulk of the contrast in an image is in the centre of k-space. Therefore the acquisition is arranged so that the echoes for the desired TE have the phase encoding gradient adjusted so that they encode the centre of k-space. In this example the encodings resulting from the third echo (the effective TE) are allocated to the centre of k-space for each TR.

Inversion Recovery Spin Echo



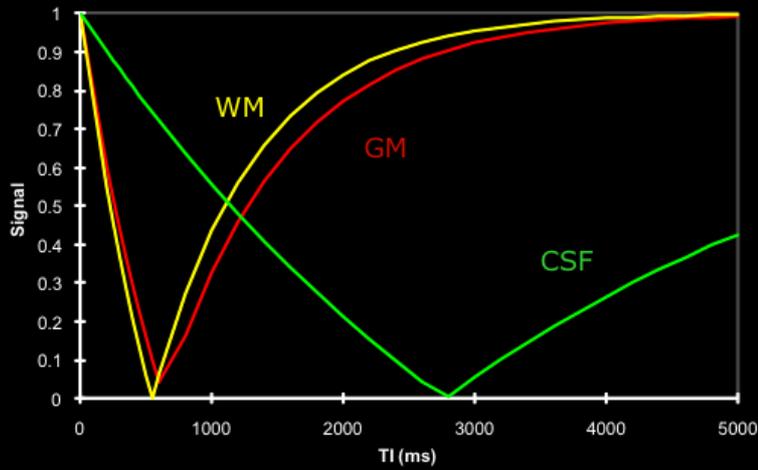
An alternative image contrast can be achieved by preceding the spin echo acquisition with an 180° pulse that inverts the z-magnetisation. We then wait for a time period known as the inversion time (TI) before playing out the spin echo, or more usually the fast spin echo readout.

Inversion Recovery - Real



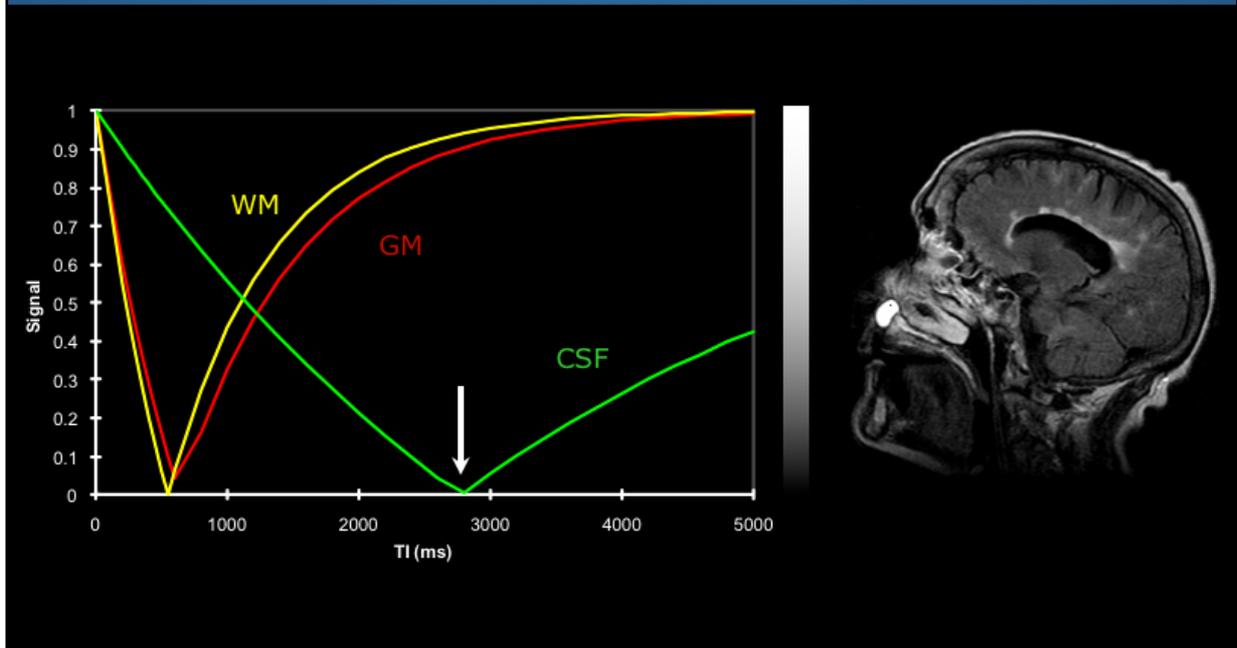
The use of an inversion pulse effectively doubles the dynamic range of the data. In this image the full signed data is achieved using a TI of 160ms. Note that zero signal (surrounding air) is represented as mid-gray.

Inversion Recovery - Magnitude



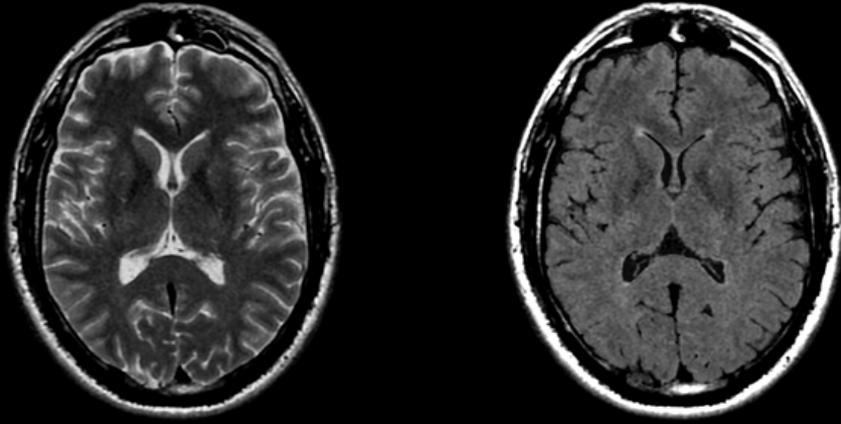
Same data as the previous slide using a TI of 160ms. However the signal magnitude has been reconstructed. Note that zero signal (surrounding air) is represented as black.

FLAIR (TI \approx 2700ms)



This is an IR sequence with a TI of approximately 2700ms. In this magnitude reconstruction the signal from CSF is nulled, providing the indication for the name of the sequence as FLuid Attenuated Inversion Recovery (FLAIR). This is a T2w image in which you can see signal hyperintensity indicative of multiple sclerosis around the ventricle. Normally these changes would be difficult to see on a conventional T2w images since there would be very high signal from the CSF in the ventricle. By knowing the T_1 of the CSF it is possible to effectively null the signal from CSF improving visualisation

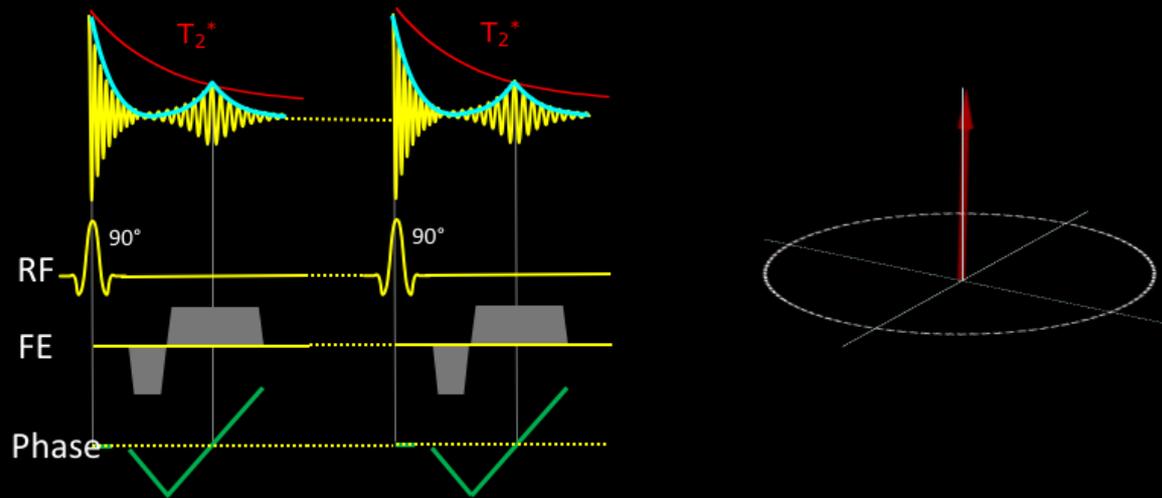
T2w-FLAIR



TR = 4000, TE = 100, TI = 1800ms

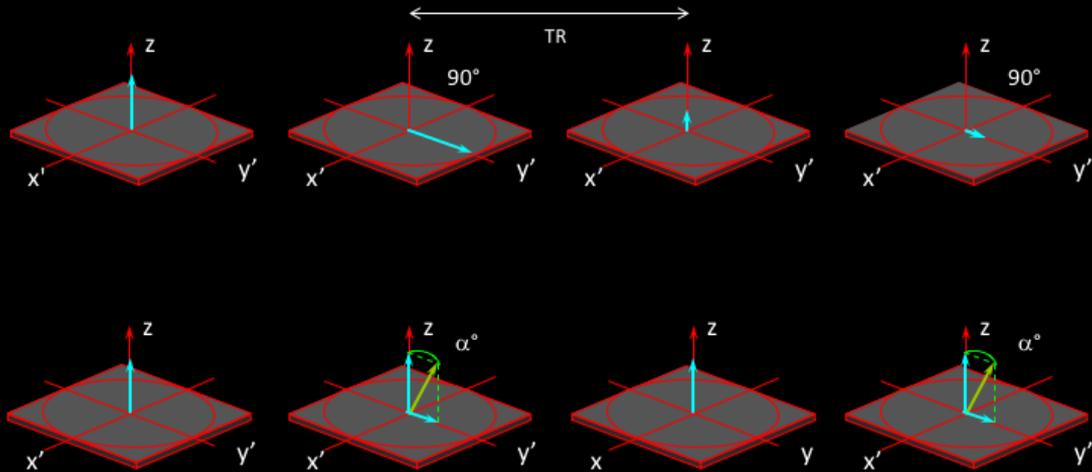
T2w image with and without IR pulse that effectively nulls the CSF in the image is the right

Gradient Echo



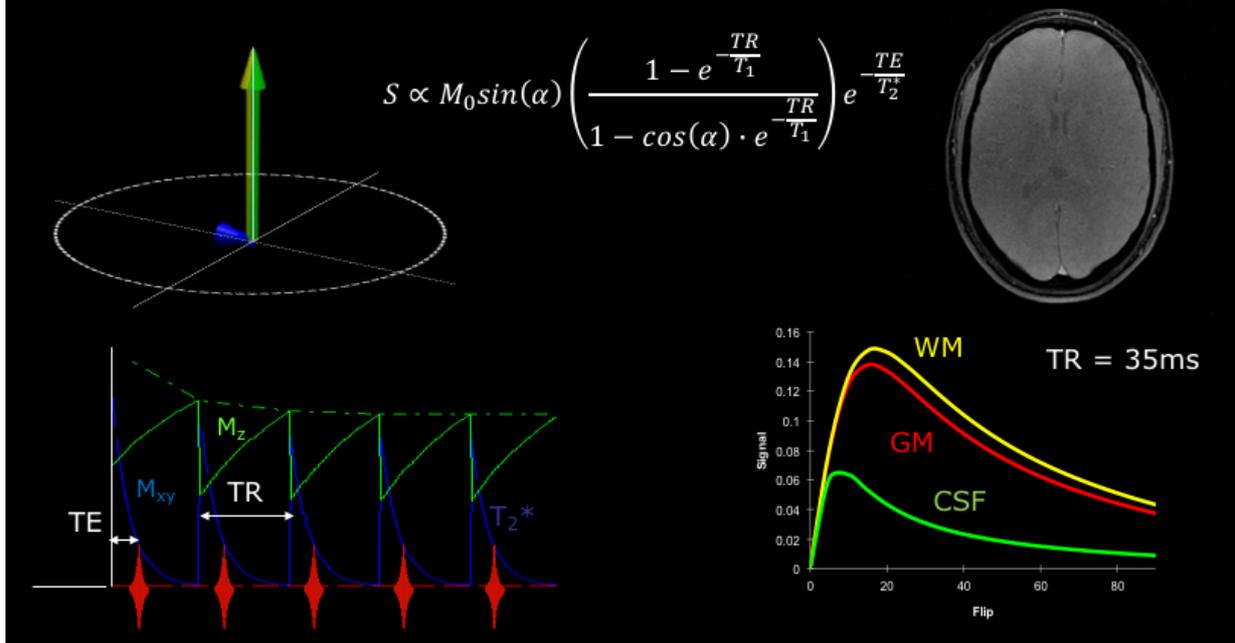
An alternative pulse sequence to spin echo is the gradient echo. This sequence does not use a 180° refocusing pulse but forcibly creates an echo through the use of the frequency encoding gradient. A negative gradient is applied that forcibly dephases the signal (note that this dephasing is in addition to the T_2^* dephasing already discussed). The polarity is then reversed and the spins rephase forming an echo. Since there is no 180° refocusing pulse the echo forms under the T_2^* envelope, rather than a T_2 envelope. A gradient echo sequence has the advantage of increasing the sensitivity of the acquisition to tissue susceptibility differences that may be enhanced in some pathologies, e.g. iron deposition. The absence of a 180° refocusing pulse means that the TE can also be made shorter.

Partial Flip Angle Excitation



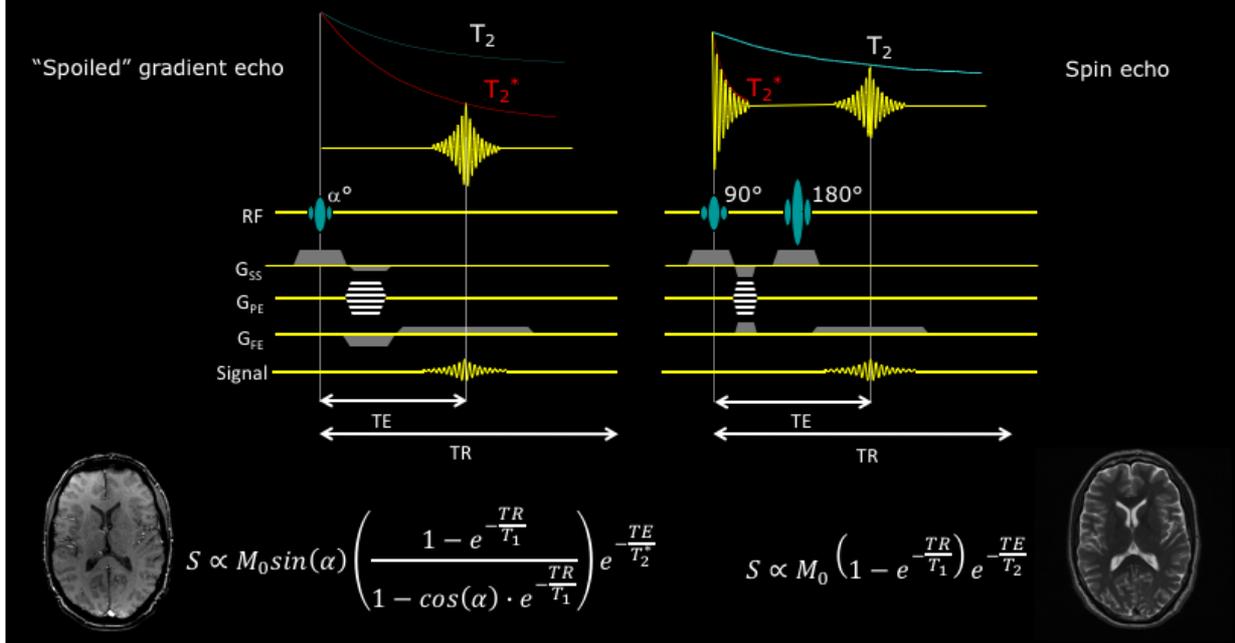
Gradient echo imaging is also often used with a short TR to reduce acquisition time. This necessitates the use of a reduced excitation flip angle to avoid signal saturation. In the top row if we use a 90° excitation with a short TR then there will be limited T1 recovery during the short TR and the available transverse magnetisation will be extremely small on subsequent excitations. Alternately on the bottom row if we tip the magnetisation by α° then as well as a component of magnetisation in the transverse plane a substantial component will remain in the longitudinal direction.

Gradient Echo Signal Behaviour



Here we can see the effect of a reduced flip angle excitation. Note that the signal behaviour is now a function of TE, TR and flip angle as shown in the slightly more complicated signal equation. The images show the effect of increasing the flip angle from 10°-90°. Note the changing contrast and decreasing SNR as the flip angle is increased.

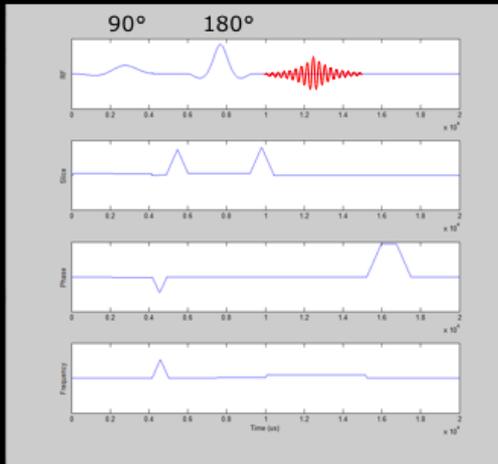
Gradient Echo vs. Spin Echo



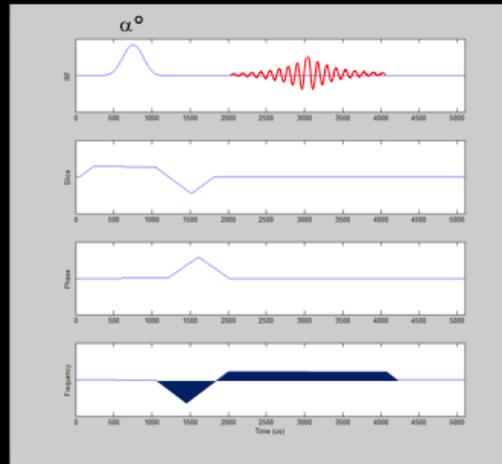
This slide shows a comparison of the signals obtained from a gradient echo and spin echo pulse sequence. Since a gradient echo sequence does not contain a 180° refocusing pulse the signal is, depending upon the TE, T_2^* -weighted, rather than T_2 -weighted as in a spin echo sequence. Furthermore the contrast in a gradient echo sequence is also a function of the excitation flip angle α° . There are various types of gradient echo sequence, this diagram shows a “spoiled” gradient echo which primarily has a T_1 -weighted appearance since TE is usually very short so that the T_2^* -weighting is minimized.

Basic Pulse Sequences

Spin echo

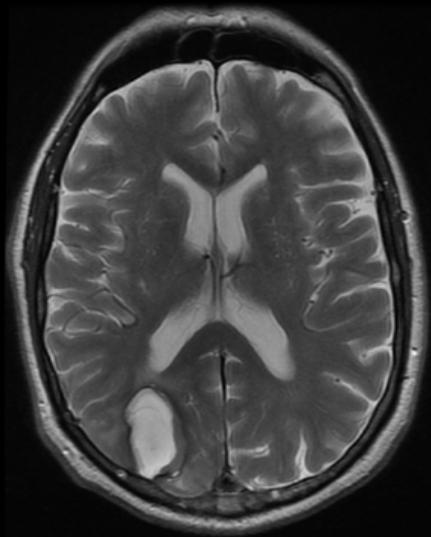


Gradient echo

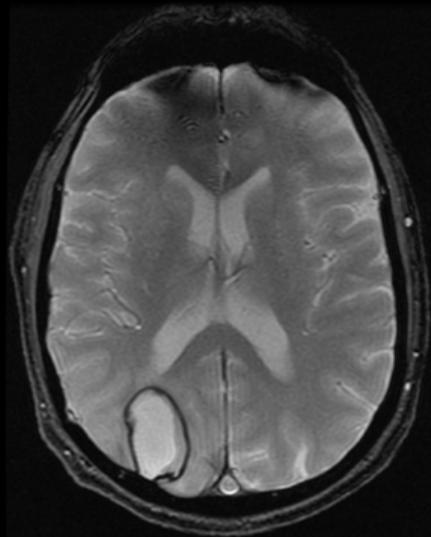


There are a wide range of pulse sequence available in MRI. This slide shows the two most basic sequences; the spin echo which we have already discussed and a gradient echo. A gradient echo does not have a 180° refocusing pulse and the echo is formed by a gradient reversal on the frequency encoding axis (shown in blue). In addition a $\alpha^\circ < 90^\circ$ excitation pulse is used so that only a component of the transverse magnetisation is tipped into the transverse plane. This allows gradient echo images to be acquired with a much shorter TE and TR than spin echo images, however the contrast can be more complex.

T2w vs. T2*w



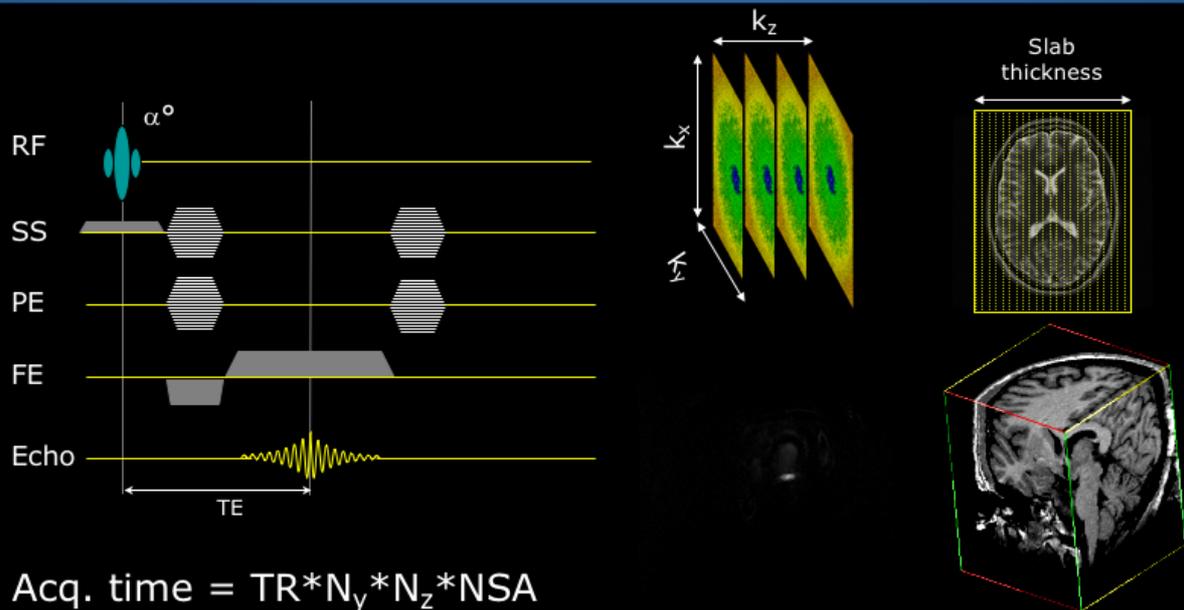
TE = 105ms TR = 3906ms



TE = 25ms TR = 800ms

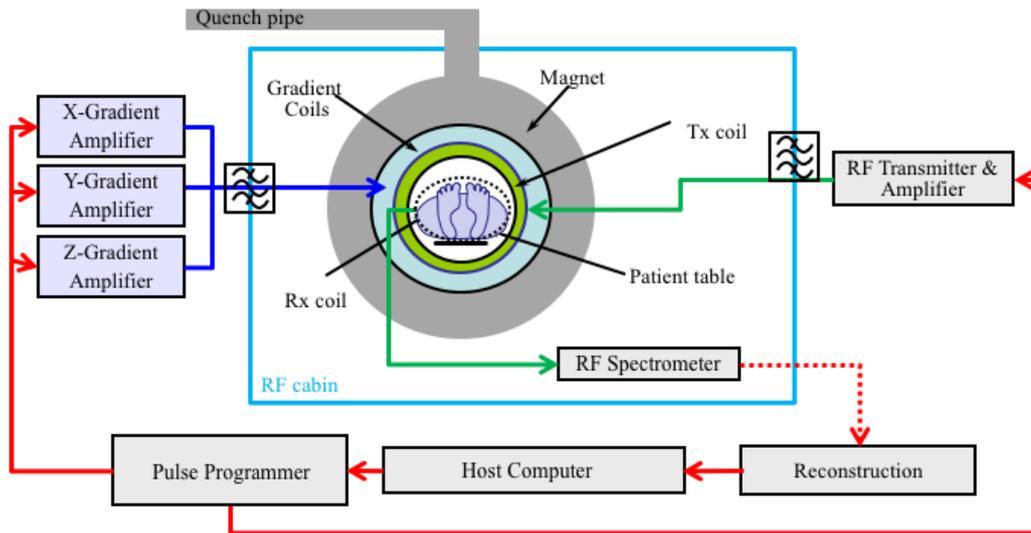
Resolving haematoma

3D Gradient Echo



One advantage of gradient echo imaging is that with a short TR truly 3D imaging can be performed. In 3D imaging a second phase encoding gradient is applied in the slice selection direction. The slice selection gradient excites a thick slab of tissue and the two phase encoding gradients together with the frequency encoding gradient encodes a 3D k-space. The images are reconstructed into conventional slices using a 3D FFT. A short TR is required since the acquisition time is given by

Hardware Overview



The diagram provides an overview of an entire MR system.

Types of Magnet

Field strengths

- Low field < 0.6T
- Mid-field ~ 1.5T
- High field ~ 3.0T
- Ultrahigh field ≥ 7 T

Technologies

- Permanent magnets
 - Neodymium Iron Boron
- Iron-cored electromagnets
 - Copper
- Superconducting magnets
 - Niobium Titanium ($T_c = 10$ K)
 - Magnesium Diboride ($T_c = 39$ K)



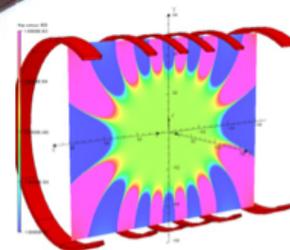
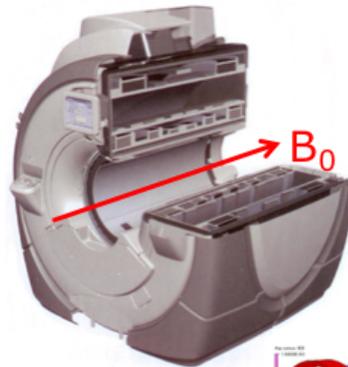
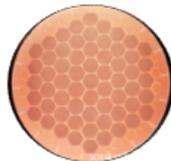
The 0.4T magnet uses permanent magnet pole pieces. The 0.6T electromagnet is an iron-cored electromagnet that weights over 100T. The MgB_2 magnet is cryogen free using two GM cryocoolers operating at 20K to mechanically keep the system operational.

Superconducting Magnet

Generates strong, highly uniform and stable static magnetic field (B_0)

- 1.5T to 11.7T and above
- Steel cryostat containing approx. 2000L liquid helium at 4.2K (-270°C)
- Cryocooler minimises helium boil-off
- Actively shielded
- $\Delta B_0(t) < 1\text{ppm}$ (40cm DSV)
- $\Delta B_0(x,y,z) < 0.01\text{ppm/hour}$

- 60km of superconducting Niobium titanium (NbTi) filaments in a copper matrix
- $I = 600\text{A}$ $E = 15\text{MJ}$



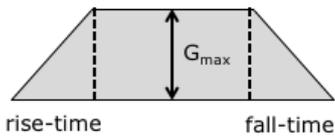
$\pm 1000\text{ppm}$

This is a cross section through a typical 3T whole body MRI scanner. The system has to generate an extremely uniform and stable magnetic field. A 3T system typically has approximately 60km of superconducting NbTi filaments in a copper matrix. The windings are housed in a steel cryostat where they are immersed in a bath of liquid helium at 4.2K at which temperature the NbTi windings are superconducting. On energising the magnet approximately 600A is injected into the windings. Boil-off of the expensive helium is reduced through a cryocooler which mechanically cools the internal radiation shields. A second set of windings outside of the main magnet windings has current circulating in the opposite direction to reduce the fringe field, this is known as active shielding.

Gradient Coils

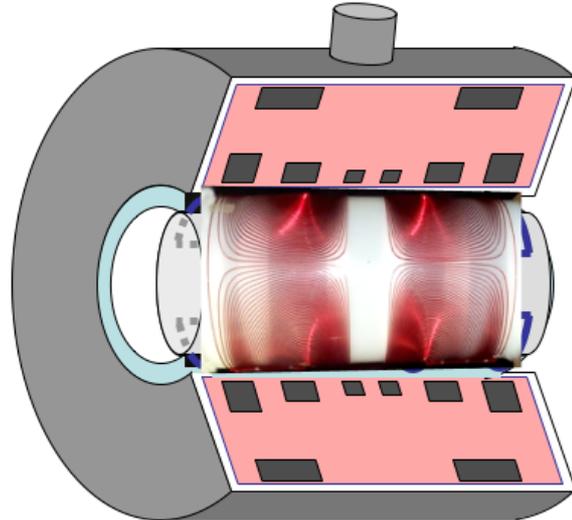
- Generate strong, linear, magnetic field variations in the z-direction, $\frac{\partial B_z}{\partial x}$, $\frac{\partial B_z}{\partial y}$, $\frac{\partial B_z}{\partial z}$ superimposed on B_0 that can be rapidly switched

- max. amplitude, e.g., 50mT/m
- min. rise time, e.g., 250 μ s
- slew rate, e.g., 200T/m/s



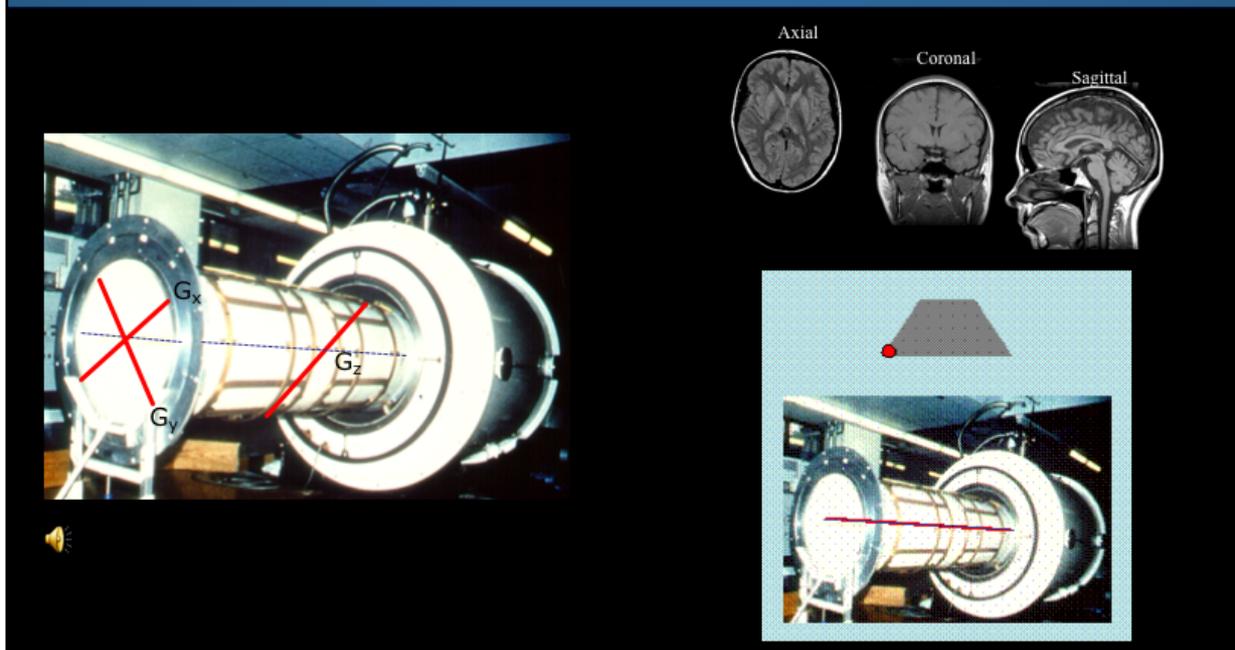
Performance specs.

- Linearity
- Eddy currents
- Heat dissipation



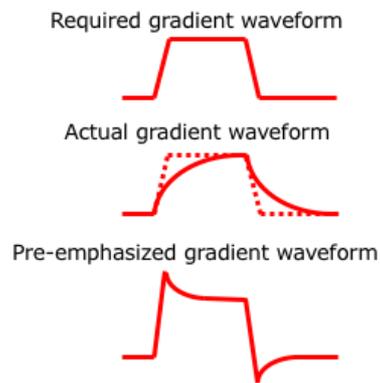
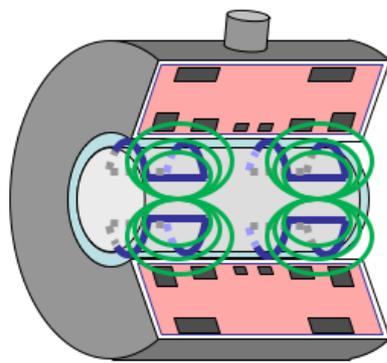
Inside the so-called room temperature bore of the magnet cryostat are positioned the gradient coils. These create the linear variations in magnetic field required for spatial localisation. The gradients must create variations along the z-direction. The z-gradient is created using a Maxwell-pair design, whilst the x and y gradients are created using a quadrupolar Golay style design. The gradients require high power amplifiers, e.g. 1.6KV @ 650A = 1MW to rapidly switch the gradients on and hold their amplitude across the bore radius. The gradient coils can generate in excess of 25KW of heat so they, and the amplifiers, are generally water cooled.

Spatial Localisation: Gradients

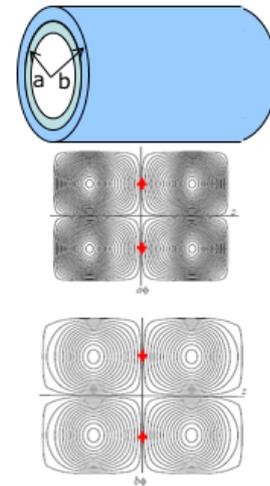


The physical G_x , G_y and G_z gradients are mapped to the logical slice select, phase encoding and frequency encoding gradients. This enables images to be obtained in any orthogonal plane or a combination of two or three allow imaging in a single or double oblique plane. The gradients physically flex when they are pulsed, like a loudspeaker, resulting in the characteristic knocking noise associated with MRI.

Eddy Currents



Actively shielded gradients

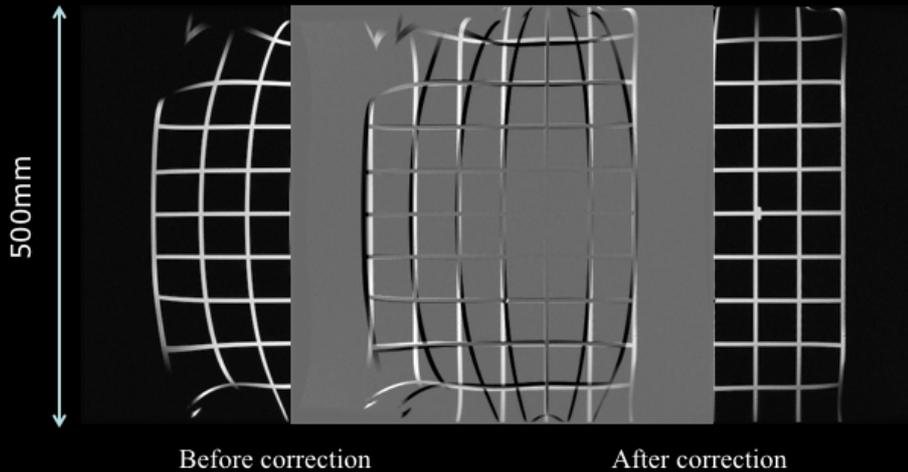


The rapid switching of the gradients induces eddy currents in nearby conducting components of the magnet cryostat. These persist for varying durations depending on the resistivity of the metal involved. The field generated by the eddy currents combines with the intended gradient field to create waveform distortions, which can result in image artifacts and signal loss.

All modern MR systems use active shielding on the gradients, very similar technology to the active shielding of the magnet coil itself. Additional “secondary” or “shield” coils surround the primary gradient coils and are driven with the opposite gradient waveform. This cancels the gradient field outside the shield coils, magnetically isolating the gradients from the cryostat so that eddy currents cannot be induced. The shield coils make the entire gradient assembly larger, reducing the free space available inside the bore, and also require more power to generate a given gradient amplitude. However, without the active shields, it is necessary to have a larger separation between the gradients and the first metal wall of the cryostat.

Although active shielded gradients induce less eddy currents, they are not perfect. The remaining effects can be minimized by *pre-emphasizing* the gradient waveforms so that, when combined with the eddy current field, the resultant is close to the ideal gradient waveform. Pre-emphasis uses extra electronic circuits that add additional voltages, with adjustable amplitudes and time constants, to the gradient driving waveform.

Gradient Non-linearity



Linear variation in the gradient field is required for accurate spatial encoding, however the gradient coils have a finite length and diameter and so they produce non-linear fields close to their edges. Linearity usually decreases fairly rapidly towards the edge of the imaging volume. The consequence of nonlinearity on an image is misplaced signal and geometric distortion. Most manufacturers make use of computer algorithms that warp the images after reconstruction to compensate for the gradient nonlinearities.

Gradient Amplifiers

- High power amplifiers are required to generate gradient pulses
- Amplifier power 1-3MW
- Gradients and amplifiers need to dissipate 25-75kW heat
- Amplifiers and gradient coils are water cooled

MGH "Connectome" gradient system
 $G_{max} = 300 \text{ mT/m}$

10,800 amps
@ 2,000V
 $\approx 22 \text{ MegaWatts}$



Los Angeles class nuclear submarine
(27 MegaWatts)



NIH Blueprints

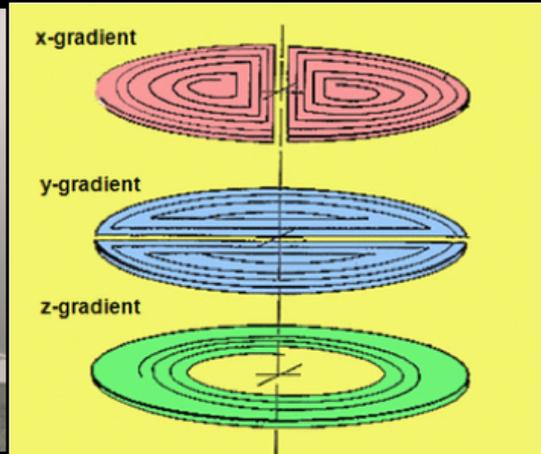
Wald, Washington DC Sept. 2014

A.A. Meltzer, Center MGH

<https://sites.udel.edu/uhf-nmr-workshop/files/2015/08/Wald-22egclw.pdf>

Gradient amplifiers operate at audio frequencies. The requirement for high gradient amplitudes means that the amplifier must be capable of producing large electrical currents through the coils. Furthermore, the requirement for short switching times means that this current must be rapidly increased from zero to the maximum and then back down. Of course, whenever you change the current flowing through a coil a 'back-emf' is generated. This is a voltage which tries to prevent the current rise in the coil; the faster the current changes, the stronger the back-emf becomes. The amplifier therefore needs to generate a sufficient driving voltage to overcome this back-emf.

Vertical Field System Gradients

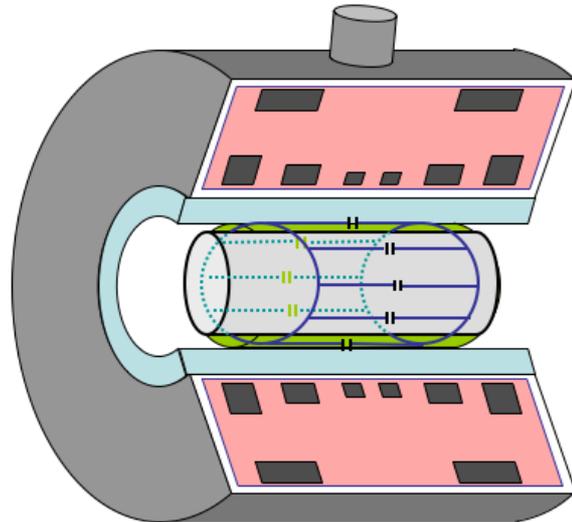


<http://mri-q.com/open-scanner-gradients.html>

Vertical field or “open” MR systems require specialised planar gradient coils. The inner arcs of the coils are primarily responsible for producing the desired gradient. By rotating these coils in plane by 90° either x- or y-gradient fields can be produced

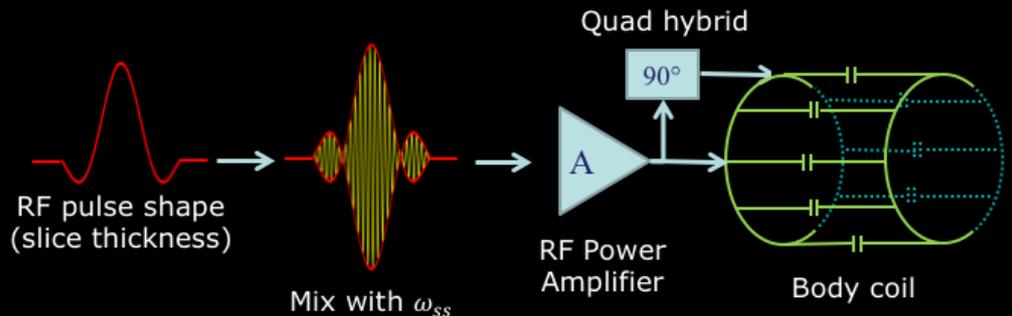
RF Transmit Coils

- Generate time-varying magnetic fields (B_1) at the required frequency (ω)
- Typ. 25kW RF amplifier
- Coils
 - LC circuits tuned to the Larmor frequency (ω_0)
 - Impedance matched (50Ω)
 - Large diameter transmit coils to maximise excitation uniformity (flip angle)
 - Separate close fitting multi-element receiver coils to maximise detection sensitivity (mV)



The purpose of the RF transmit coil is to create the circularly polarised time-varying B_1 field at right angles to B_0 . A large diameter birdcage “body” coil is generally used since this creates the most spatially uniform B_1 field. The coil is a resonant circuit tuned to the Larmor frequency for the given field strength and impedance matched to the RF power amplifier.

RF Transmission



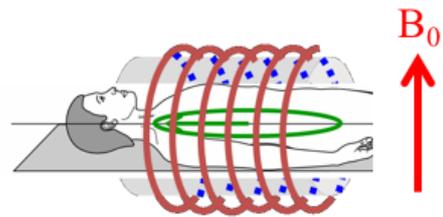
RF transmission involves mixing the digitally generated RF pulse envelope (which dictates the bandwidth of the excitation pulse) with the slice select frequency. This analogue waveform is amplified and applied in quadrature to the birdcage body coil in order to generate a circularly polarised RF that rotates in the same sense as the precessing nuclear magnetisation.

The main transmitting coil is usually the body coil, which surrounds the entire patient. This is usually built into the scanner bore and is not generally visible. Since this coil is large it has a very uniform transmission field, but this also means that it is not particularly sensitive if used as a receiver coil. In some systems other coils, e.g. head or knee, may also be used for transmission, in which case less power is required to flip the magnetization, but excitation uniformity may be sacrificed.

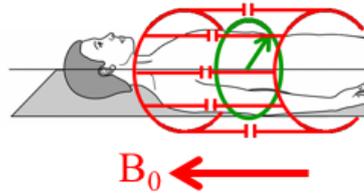
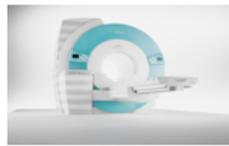
Types of RF Transmit Coil

RF coil design depends upon the orientation of B_0 compared to the subject

Vertical field magnet
- solenoid coils



Horizontal field magnet
- birdcage coil



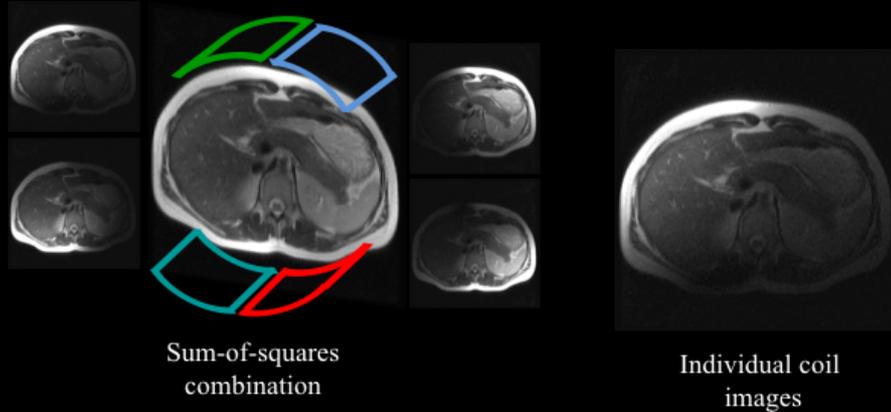
Vertical or horizontal field systems can use simple solenoids to create B_1 since B_0 is orthogonal. In superconducting magnets where the field is horizontal the coils need to be of the birdcage design.



RF Receive Coils

Example multi-channel coils

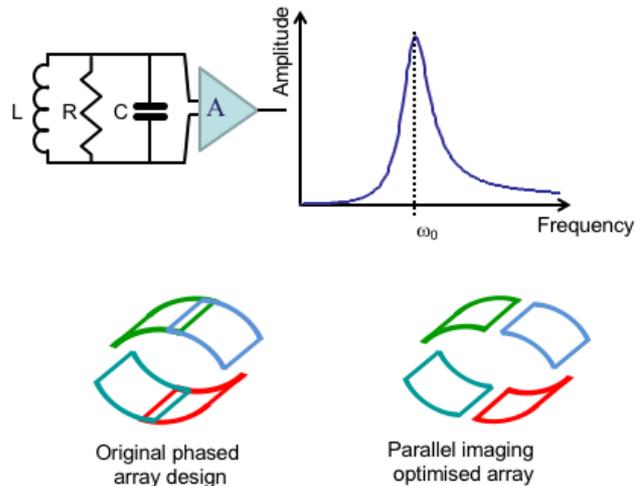
- Head
- Neck
- Spine
- Knee
- Foot/ankle
- Wrist
- Shoulder
- Cardiac
- Breast
- Torso
- Abdominal
- Endo-rectal
- Flexible



Receiver coils generally comprise arrays of smaller individual coils or elements. The smaller the coil the better the signal-to-noise ratio, however each individual coil element only has a limited depth penetration. Therefore, arrays of coils are used to achieve the desired coverage. The coils are electronically decoupled from each other so that they don't just appear as a single "large" coil. The images from individual coils are independently reconstructed and then summed together to create the final image.

Coil Basics

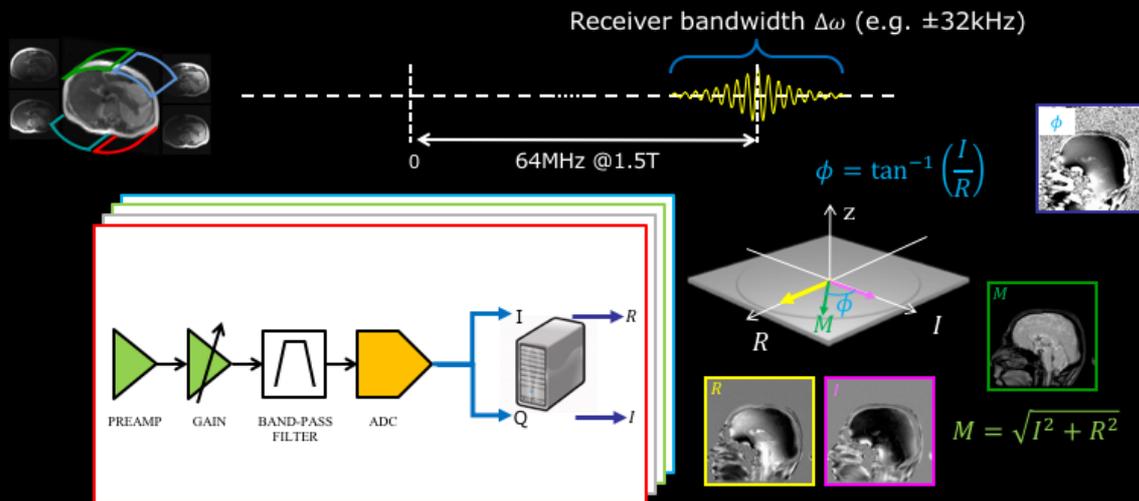
- Coils are resonant tuned circuits
- A coil has a depth sensitivity approx. equal to its diameter
- Arrays are combinations of individual coils that cover a large area/volume but do not interact
- Array geometries are optimised for parallel imaging



Both transmit and receive coils are tuned circuits. The majority of receive coils in modern systems consist of multiple elements, each acting independently. These coils are formally known as “phased array” coils, but since they are now so common, we often just use the word “coil”.

These coils need to be designed very carefully so that the individual elements do not interact with each other, a phenomenon known as coupling which reduces SNR in the images. One way of effectively ‘decoupling’ one element from its neighbors is to geometrically overlap the coils in a particular way. Each element is connected to an entirely separate preamplifier and receiver which has the advantage that the noise in each receiver is completely different, i.e. uncorrelated, resulting in a higher SNR in the final image. However recent developments in very low impedance preamplifiers built into coils makes the designs much more flexible. Developments such as parallel imaging used to accelerate MRI acquisitions require different spatial sensitivities for the individual coils.

Receiver (x N-Channels)



The MR signal only contains a narrow frequency range of interest $\Delta\omega$, embedded in or carried by the Larmor frequency ω_0 (with an offset due to the frequency encoding). Mathematically this is shown as $\cos(\omega_0 + \omega_{FE} \pm \Delta\omega)$. In earlier MR systems, it was not possible to digitize directly at the Larmor frequency, so the MR signal had to be demodulated down to a lower frequency (e.g. 125 kHz) before converting to a digital signal. However, analog-to-digital converters are much faster now and all modern system now use direct digital receivers. These ADCs operate at very high frequencies, 10s of MHz, and allow 16 or 32 bit digitization. This also has the advantage of minimizing image artifacts caused by drift in the older analog receiver circuitry. Each coil element is connected to its own receiver. Since the signals are detected in quadrature the reconstructed MR data is complex. We primarily reconstruct magnitude images but we can also use gradients to manipulate the phase of the MR signal which can be used to encode information such as the velocity of blood flow.

Signal-to-noise Ratio (SNR)

$$SNR \propto \frac{\Delta x \cdot \Delta y \cdot \Delta z \cdot \sqrt{N_x \cdot N_y \cdot NSA}}{\sqrt{RBW}}$$

Remember that some parameters are interdependent e.g. changing the matrix size will change the acquisition time as well as the pixel size

Some SNR relationships in MRI

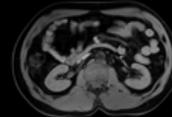
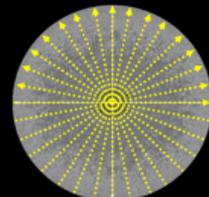
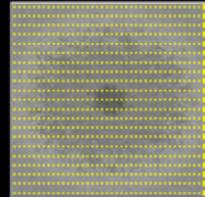
Reconstruction

› Role:

- Manage raw data
- Reconstruct images
- Inline post processing

› Performance specifications:

- Raw data capacity
- Reconstruction algorithms
- Reconstruction speed
- Post processing methods



Future Hardware Developments

▸ Magnets

- Low cryogen volumes, e.g. 10L (no quench pipe)

▸ Gradients

- Higher performance with wider bore
- Head-only systems

▸ RF

- Higher channel counts
- Lighter more flexible coils
- Integrated high-order shims
- On-coil digitisation

▸ Reconstruction

- More sophisticated acquisition and hence reconstruction algorithms
- Memory size and speed improvements

▸ Combined/hybrid modalities

- PET/MR
- MR-Linac

MR Safety

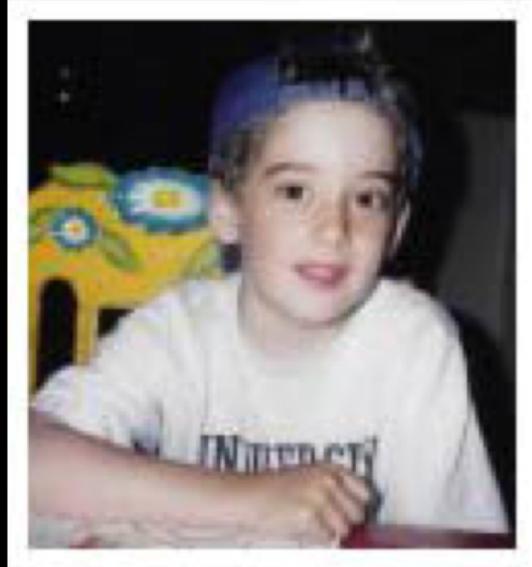
Wheelchair flies across hospital room

A hospital is facing a £20,000 bill to repair damage caused by a wheelchair which was flung across a room by powerful magnets in an MRI machine.

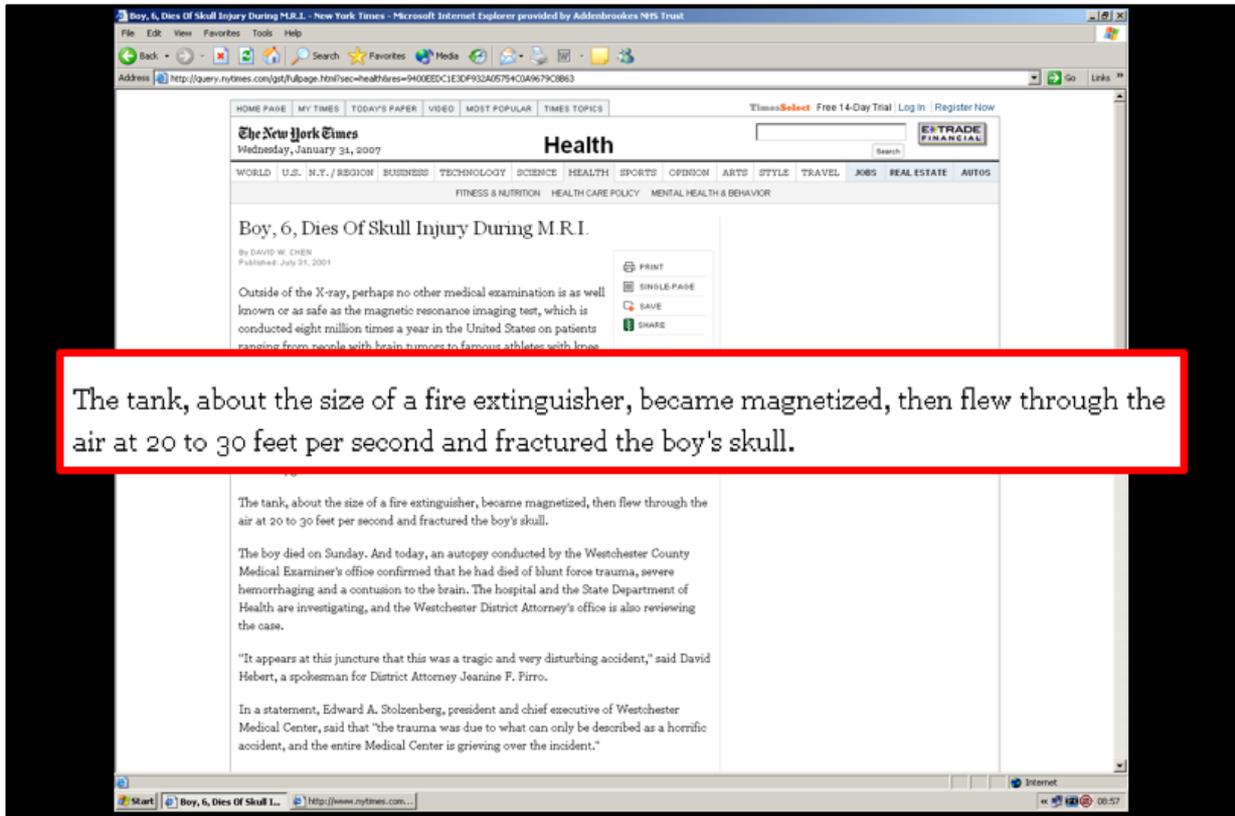


Michael Colombini

- › Born 1995 Croton-on-Hudson NY
- › 2001 suffers a “nasty fall” at home
- › CT shows benign brain tumour
- › Tumour resected
- › Post-op MRI requested
- › Michael sedated, taken into MRI scanner
- › Sats start dropping ...



This was a sentinel event in the world of MR safety



Oxygen Cylinder



September 2010

\$2.9 Million Settlement Closes Colombini MRI Death Case

This week the settlement documents were released — closing the chapter on the lawsuit that arose from the seminal event in MRI safety, the 2001 oxygen tank fatality of then-six-year-old Michael Colombini.

Nearly nine years after the accident, the lawsuit was settled for \$2.9 million, a settlement that was likely both diminished by, and made possible by, a pre-trial motion which excused GE Healthcare as a defendant to the suit.

Applications Specialist

“First day of applications training... apparently I went to the back of the magnet where there was a laptop trolley stand in the corner... Somehow it was attracted to the magnet and hit me in the face”

“I’m fine now. I see 50% in my right eye, cannot smell anymore and have 11 titanium pieces in my face”



zö 

India - November 2014

Indian technician awarded compensation after accident

By AuntMinnie.com staff writers



February 6, 2017 -- A former employee of Tata Memorial Hospital in Mumbai, India, has received compensation from GE Healthcare more than two years after being seriously injured in an MRI accident.

Technician Swami Ramaiah was caught between an MRI scanner and a fellow employee who brought an oxygen cylinder into the MRI suite in the November 2014 incident, according to a report in the *Mumbai Mirror*. Ramaiah was paralyzed below the waist and had injuries to his bladder and kidneys, while the other employee had a fractured shoulder.

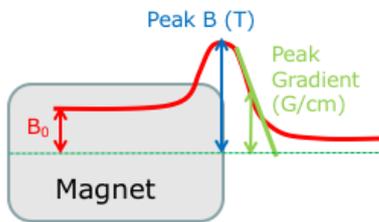
The plaintiffs alleged that the MRI scanner's off switch had been disabled by GE, pinning the men to the machine. The men were freed from their situation hours later after an engineer from GE arrived and deactivated the magnet, according to the report.

Ramaiah received 1 crore rupees, or about \$215,000, in compensation for the accident, according to the article. He is now able to walk unaided and returned to work in May 2015, but it reportedly took until last month before he could return to the MRI suite due to lingering traumatic aftereffects.

In February 2015, GE issued a recall of its MRI scanners "in response to a single safety incident in India" that was a "case of human error on site." GE asked its customers to perform a five-minute check to ensure that the scanners' magnetic rundown unit had not been disconnected as a part of postinstallation service. The unit is essentially an "emergency off switch" for the MRI magnet, according to GE.

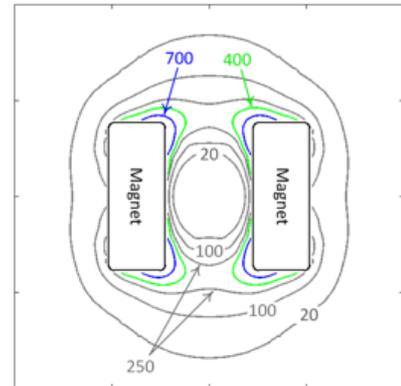
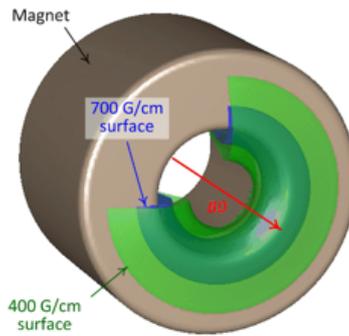
oxygen
net room
staff trapped
own unit

B₀ Spatial Gradient



100 G/cm = 1.0 T/m

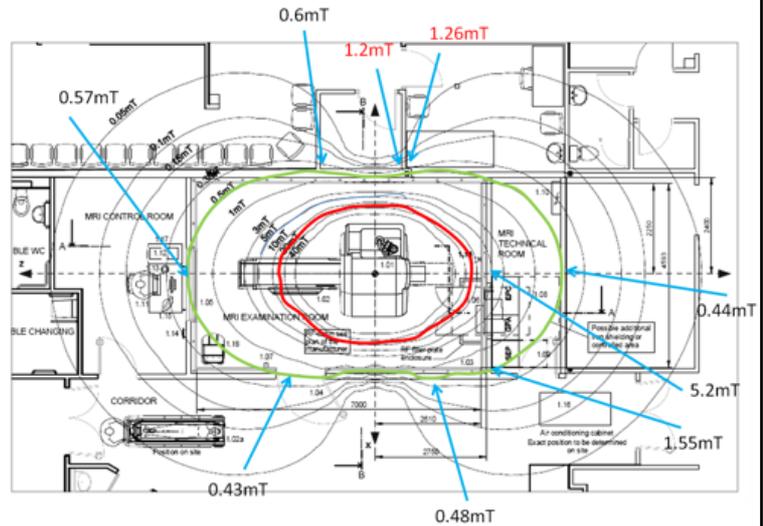
$$F \propto \chi V B \frac{dB}{dz}$$



In this example, there is no path into the magnet that does not pass through the "entry SG" contour of 250 G/cm (2.5 T/m)

Fringe Field

- ▶ Restricted access to 0.5mT/5G field contour
- ▶ UK MHRA
 - MR Environment
- ▶ ACR
 - Zone III



Quench

- A superconducting magnet holds around 2,000 litres of liquid helium (-270°C)
- If the magnet ceases to be superconducting the huge energy stored (15MJ) is converted into heat
- The liquid helium is vaporised with a 754:1 expansion ratio (1.5M litres)
- The very cold gas is vented to the outside through the quench pipe
- In a double fault situation the gas could theoretically fill the magnet room
- Risk of cold contact burn, asphyxia and explosion risk from liquified oxygen



Gradients

- ▶ 100 times weaker than the static magnetic field
- ▶ Switch about 0-10kHz
- ▶ Peripheral nerve stimulation
 - ♦ From tingling sensation to painful muscle spasm
 - ♦ Avoid creating large loops
- ▶ Acoustic noise
 - ♦ Can exceed 100dBA
 - ♦ Hearing protection recommended



Radiofrequency Magnetic Fields

- ▶ 10,000 times weaker than the static magnetic field
- ▶ Switch about 60-130MHz
- ▶ Main effect is tissue heating, like a microwave oven or mobile phone
- ▶ Mobile phone: 2W
- ▶ Microwave: 1,000W
- ▶ MRI: 30,000W
- ▶ Whilst tissue heating is strictly controlled, anything electrically conducting can heat up



Specific Absorption Rate (SAR)

- Faraday's law gives $EMF \propto \frac{dB}{dt}$
- Electric fields produced in conducting tissues will drive electric currents and result in tissue heating
- For a volume conductor with induced current density \mathbf{J} and induced electric field \mathbf{E} the power is given by $P = \mathbf{J} \cdot \mathbf{E} = \sigma E^2$, where σ is the conductivity (Sm^{-1})
- The specific absorption rate (SAR) (Wkg^{-1}) is defined as $SAR = \frac{\sigma |E|^2}{\rho}$, where ρ is the tissue density
- Integrating over a spherical volume of radius R

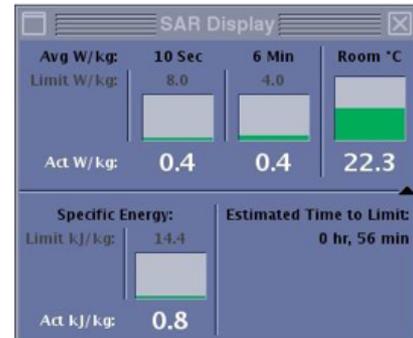
$$SAR = \int_{r=0}^R \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{\sigma |E|^2}{\rho} r \, dr d\theta d\phi$$

$$SAR = \frac{\pi \sigma \omega_0^2 B_1^2 R^5}{15}$$
- Assuming uniform conductivity!
- Importantly,
 - SAR increases with B_0^2 i.e. static field strength squared
 - SAR increases with B_1^2 i.e. flip angle squared
 - SAR increases with patient size

Less than 1% of the energy dissipated is used to tip the nuclei. The vast majority produces nothing but undesirable heating

RF Exposure Limits (IEC 60601-2-33)

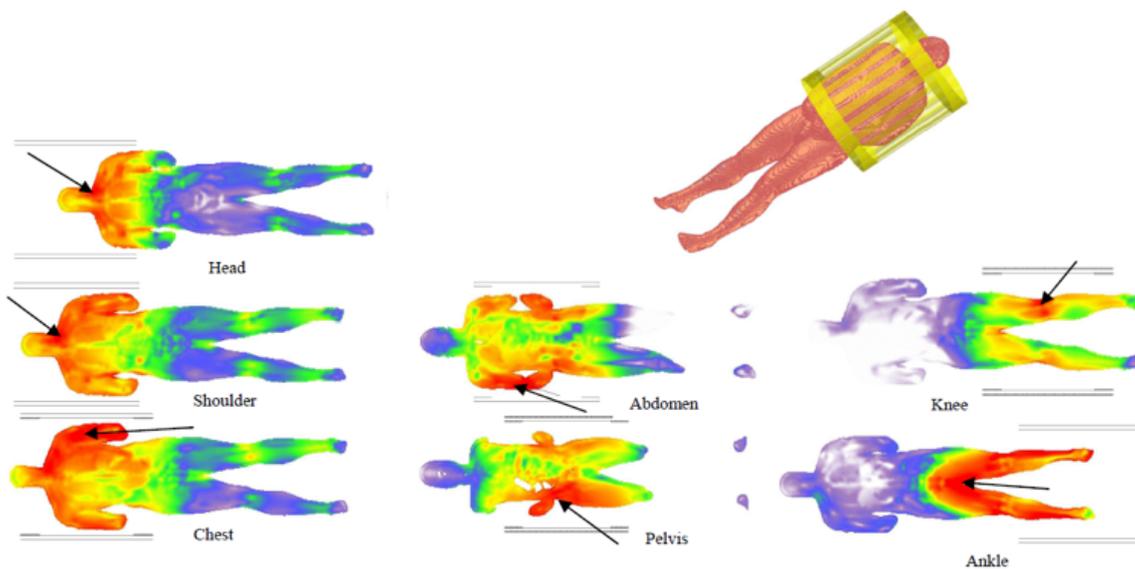
	Whole body SAR	Partial body SAR	Head SAR	Local SAR (a)		
Body Region →	whole body	exposed body part	head	head	trunk	extremities
Operating Mode ↓	(W/kg)	(W/kg)	(W/kg)	(W/kg)	(W/kg)	(W/kg)
Normal	2	2 - 10 (b)	3.2	10 (c)	10	20
1st Level Controlled	4	4 - 10 (b)	3.2	20 (c)	20	40
2nd Level Controlled	>4	>(4 - 10) (b)	>3.2	>20 (c)	>20	>40
Short duration SAR	The SAR limit over any 10 s period shall not exceed two times the stated values					



The SAR is averaged over a 6 minute period

Manufacturers empirically determine the energy absorbed within the subject.
 For example, a 1ms hard 180° RF pulse has a B_1 of 11.7 μ T

Simulation of Patient Loading on SAR Hotspots at 3T



Chen et al http://eta2.bio.cmu.edu/ISMRM/ISMRM_2010_Stockholm/files/3879_5934.pdf

RF Burn Example

BURNS 35 (2009) 294–296



ELSEVIER

available at www.sciencedirect.com



journal homepage: www.elsevier.com/locate/burns



Case report

MRI induced fourth-degree burn in an extremity, leading to amputation

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Metallic Microfibre Clothing

Published March 1, 2012 as 10.3174/ajnr.A2827

TECHNICAL NOTE

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Invisible Metallic Microfiber in Clothing Presents Unrecognized MRI Risk for Cutaneous Burn

SUMMARY: We report a case of a thermal burn that occurred during MR imaging likely caused by invisible silver-embedded microfibers in the fabric of an undershirt. As the prevalence of fabric containing nondetectable metallic microfiber increases in athletic and "tech" clothing, the importance of having patients change into safe facility-provided garments before MR imaging is emphasized.

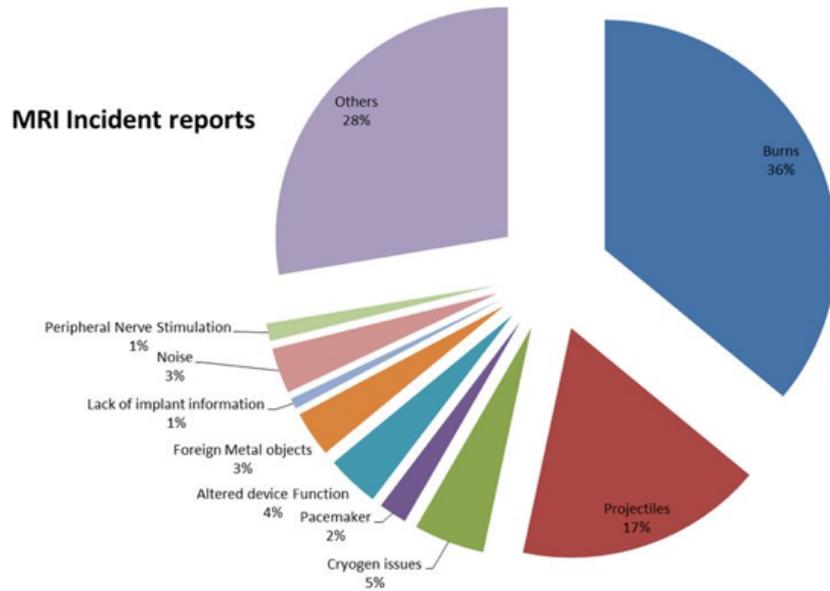
ABBREVIATIONS: ACR = American College of Radiology; SAR = specific absorption rate; SMF = silver microfiber



Fig 1. A linear erythematous blistering eruption is noted on the patient's right flank minutes after completion of the MR imaging of her brain and spine.



MHRA Incident Reports



Safety Questionnaire

Consent Form

Magnetic Resonance Imaging (MRI):

PATIENT NAME: _____ Weight _____
 Height: _____ (in or ft)

1. MRI SAFETY QUESTIONNAIRE

Do you have any of the following devices in/on your body?

- Cardiac/Heart Pacemaker/pacing wires or implanted
- Artificial heart valve
- Electrical stimulator for nerves, brain or bone?
- Implanted infusion or drug pump?
- Coils, filters, shunts or stents?
- Aneurysm clips?
- Ocular (eye) implant?
- Cochlear (ear) implant?
- Have you ever had metal in your eyes or worked
- Mechanically or electronically activated metal
- Bullets, shrapnel or other pieces of metal in yo
- Medicated skin patches (eg pain relief, hormo

The following devices can affect the quality of M
 We need to know if any of these items are prese

- Dental work: dentures, or dental plate
- Hearing aid
- Metal joints/joint replacement, pins, plates
- Tattoos
- Body piercing

Have you ever had any surgery?
 If yes, please list: _____

For females of childbearing age:
 • Is it possible that you may be pregnant during contrast

Consent Form

Magnetic Resonance Imaging (MRI):

2. MRI CLINICAL INFORMATION QUESTIONNAIRE

What problem(s) brought you to the doctor/health professional that resulted in this MRI scan being ordered?

What do you think might have caused the problem and when did it start?

Have you had any surgery / treatment on the body region that we are scanning today? Yes No

If so, please list:

DATE	TYPE OF SURGERY / TREATMENT	NAME OF SURGEON / HEALTH CARE PROVIDER

Have you had any of the following tests done that are relevant to your current medical condition?

	WHEN	WHERE	RESULT
X-ray	Yes <input type="checkbox"/> No <input type="checkbox"/>		
Ultrasound	Yes <input type="checkbox"/> No <input type="checkbox"/>		
MRI	Yes <input type="checkbox"/> No <input type="checkbox"/>		
CT	Yes <input type="checkbox"/> No <input type="checkbox"/>		
Other	Yes <input type="checkbox"/> No <input type="checkbox"/>		

Please circle the area of pain/discomfort on the drawing to the right, indicating symptoms with the below letters.

Key: D: Dull ache
 S: Sharp pain
 N: Numbness
 T: Tingling



Example Clinical Applications

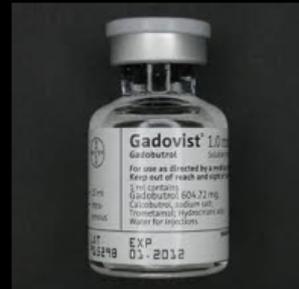


Clinical Applications

- Neurology – stroke, multiple sclerosis, tumours
- Oncology – diagnosis, staging, treatment response
- Musculoskeletal – trauma, degeneration
- Cardiovascular – heart and blood vessel morphology and function
- Infectious diseases and inflammation
- Congenital/anatomical defects

Gadolinium Based Contrast Agent (GBCA)

- ▶ Chelate of Gadolinium (Gd^{3+})
- ▶ Seven unpaired electrons in outer shell
- ▶ 2,000x larger magnetic moment than proton
- ▶ Increases T_1 (and T_2 relaxation) of nearby water molecules
- ▶ Demonstrates T1w hyper-enhancement of perfused lesions



Contrast Agent Relaxation

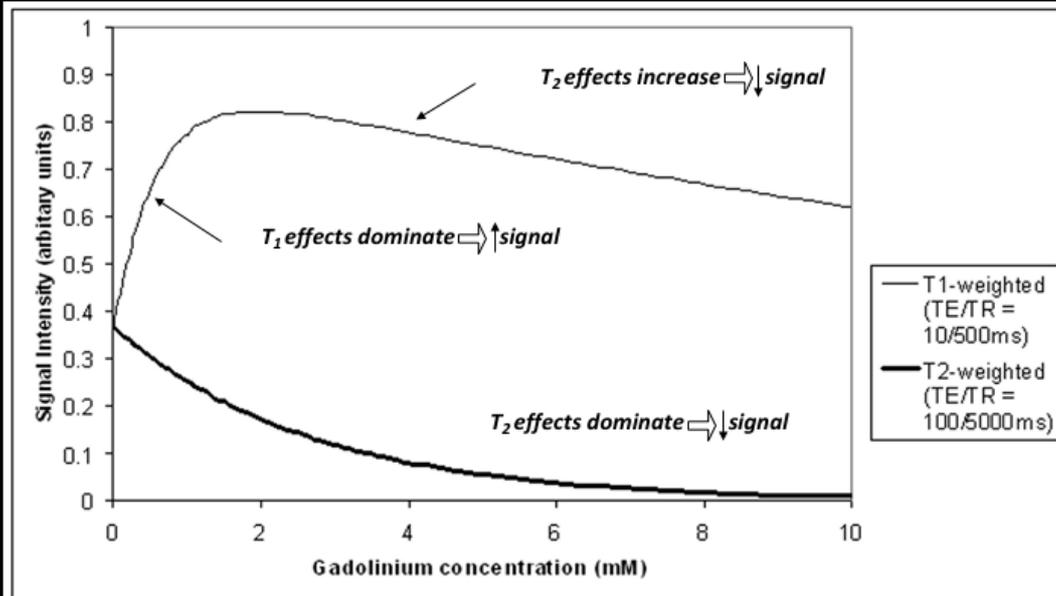
$$\begin{aligned}(1/T_1)_{\text{post}} &= [\text{Gd}] R_1 + (1/T_1)_{\text{pre}} \\ (1/T_2)_{\text{post}} &= [\text{Gd}] R_2 + (1/T_2)_{\text{pre}}\end{aligned}$$

[Gd] is the concentration (mM L^{-1})

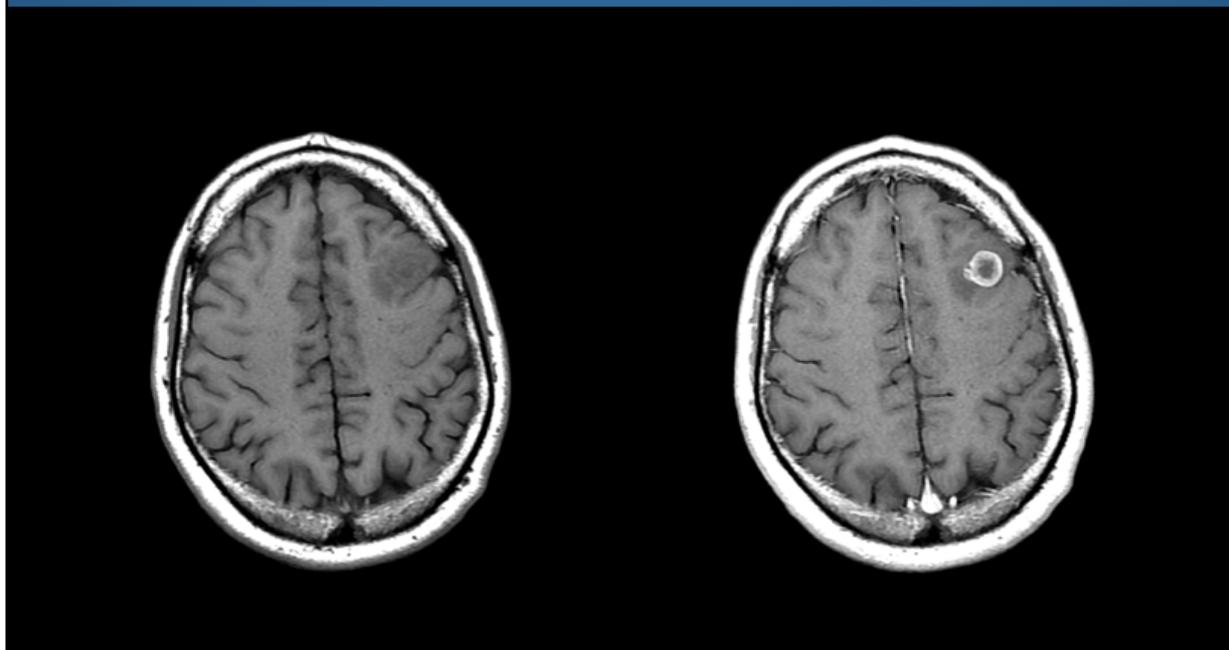
R_1 and R_2 are the T_1 and T_2 relaxivities of the contrast agent ($\text{L mM}^{-1} \text{s}^{-1}$)

[N.B. These are dependent upon B_0 , temperature, surrounding material, e.g. water, plasma, blood]

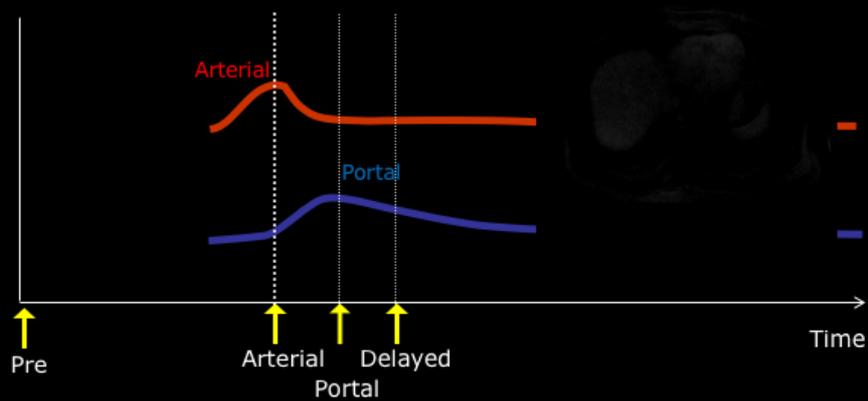
GBCA Contrast Behaviour



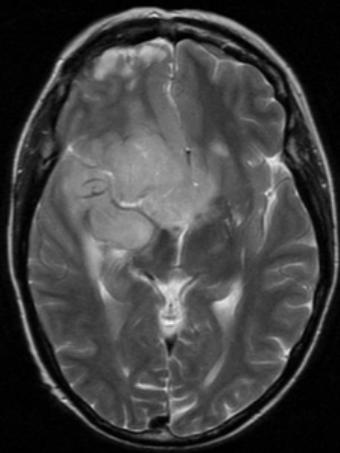
Pre + Post Contrast T1w



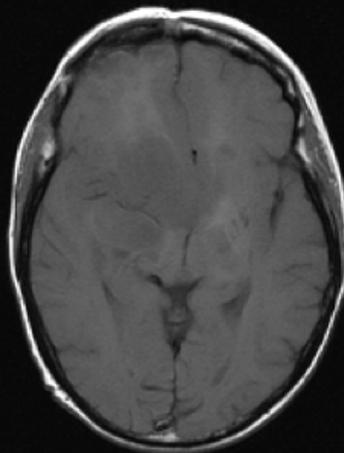
Contrast Enhanced Liver imaging



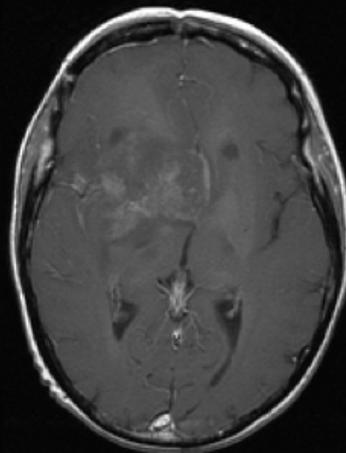
Primitive Neuroectodermal Tumour



T2w

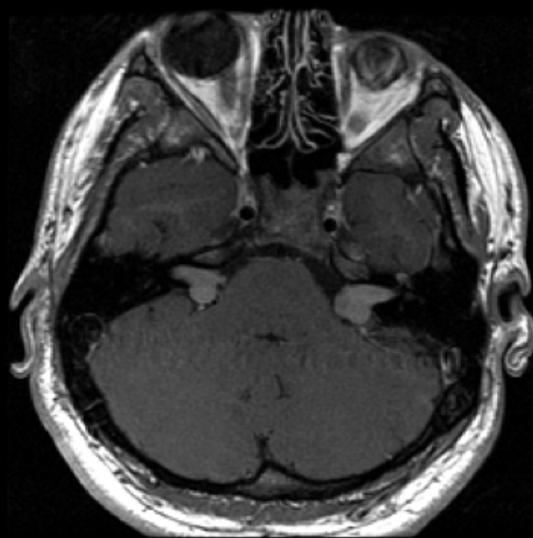


T1w

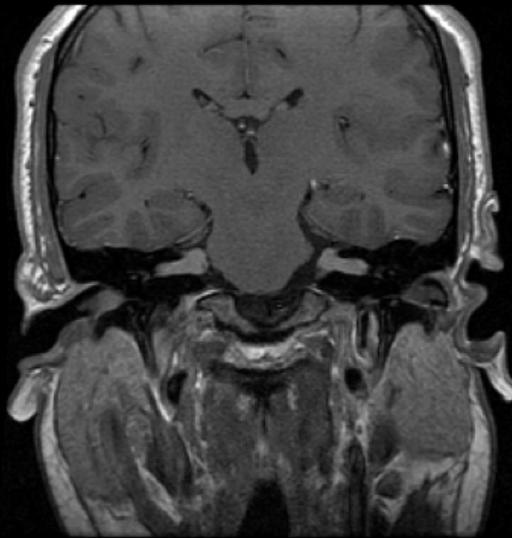


T1w + C

Acoustic Neuroma

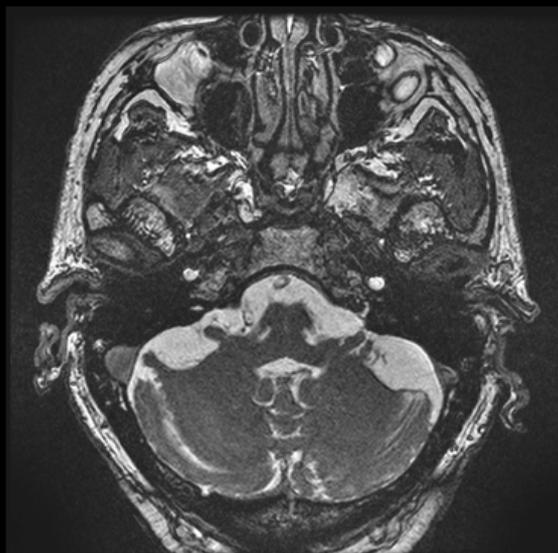


T1w + C

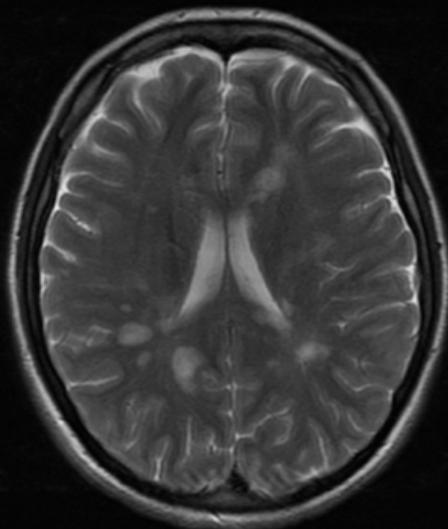


T1w + C

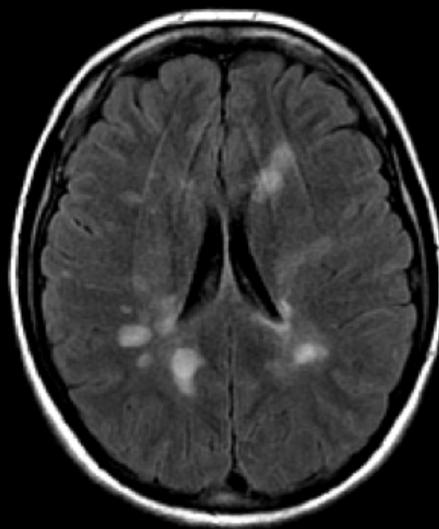
Semi-circular Canals



Multiple Sclerosis

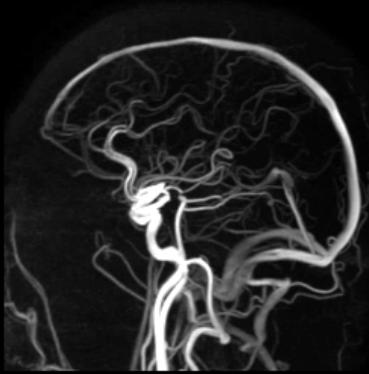


T2w



FLAIR

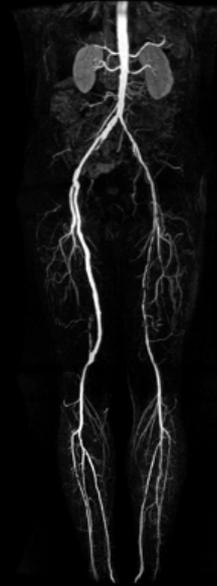
MR Angiography



Phase contrast MRA



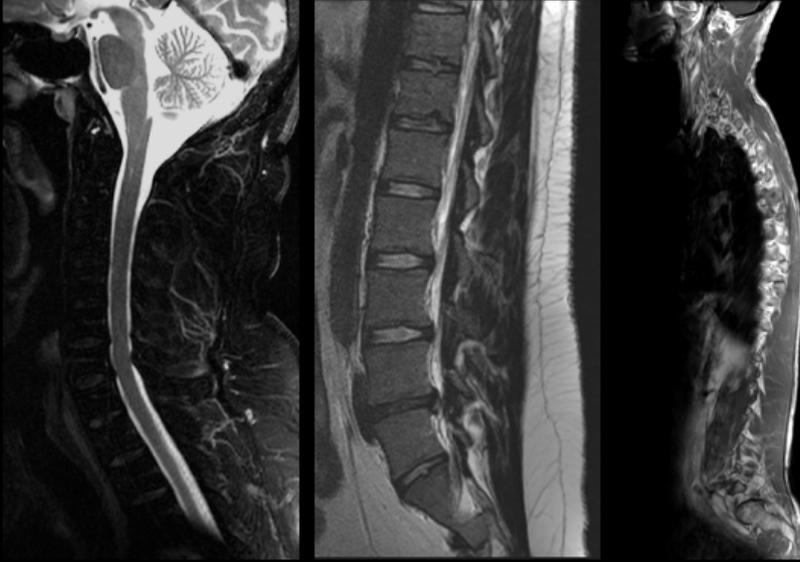
Dynamic contrast enhanced MRA



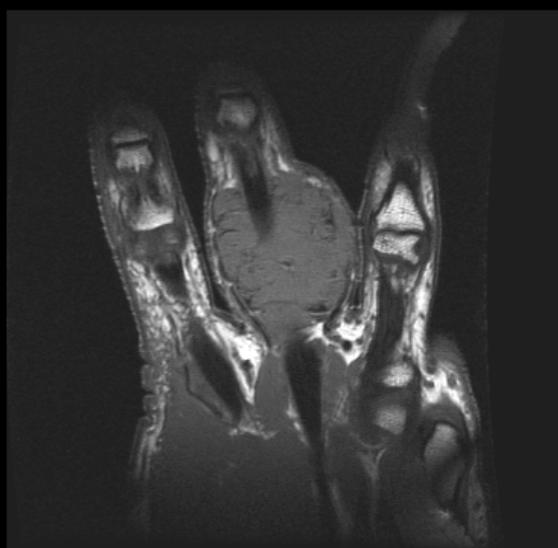
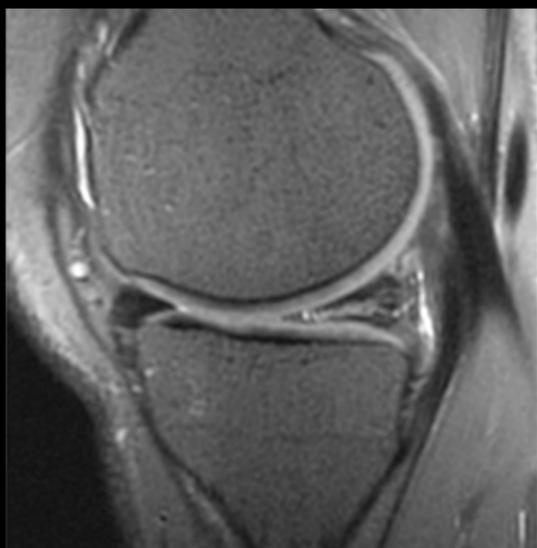
www.1450wv.75

Whole-body contrast enhanced MRA

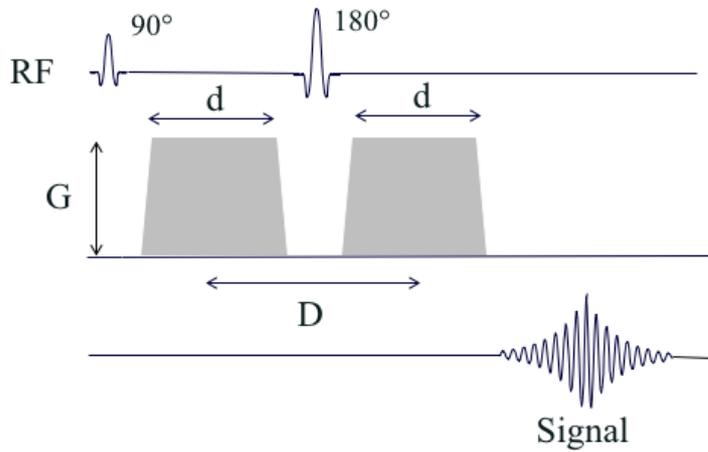
Spine



Musculoskeletal



Diffusion Weighted Imaging

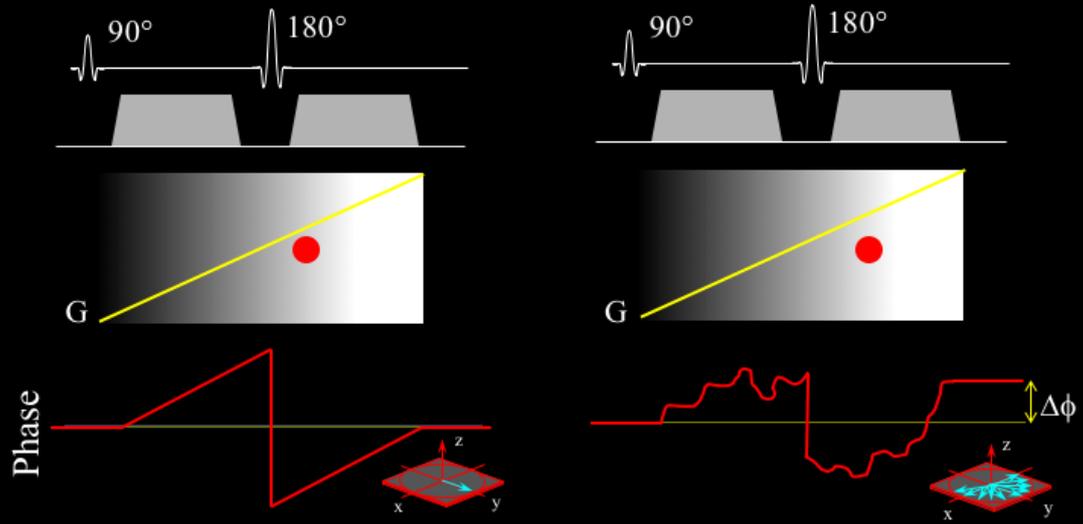


$$\frac{S}{S_0} = \exp\left[-\left(\gamma^2 G^2 \delta^2 \left(\Delta - \frac{\delta}{3}\right)\right) D\right]$$

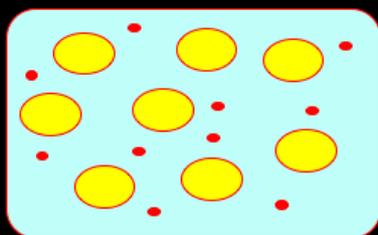
$$\frac{S}{S_0} = \exp(-bD)$$

b is the gradient factor in s mm^{-2}

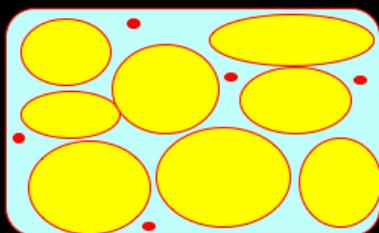
Diffusion in MRI



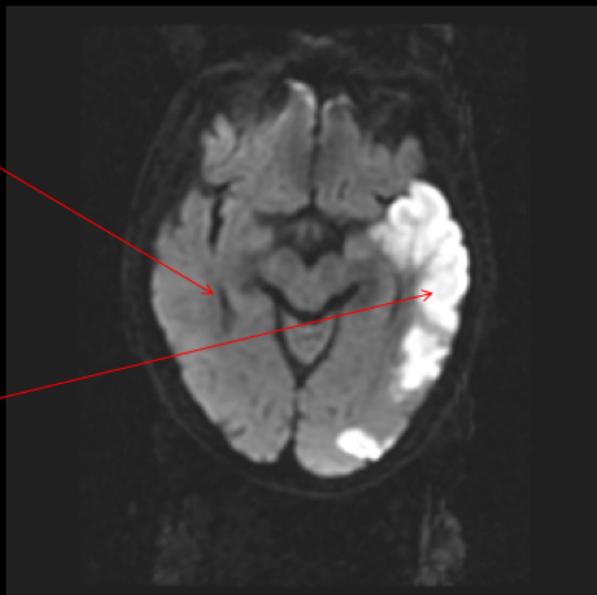
Diffusion Weighted Imaging



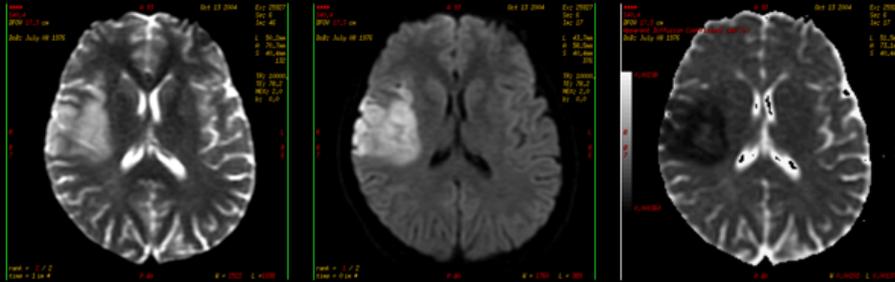
Freely diffusing water
- dark



Restricted diffusion
- bright



Quantitative Apparent Diffusion Coefficient (ADC)



b=0 (T₂ weighted)

b=1000

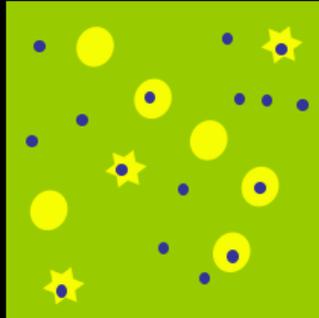
ADC map

$$S_{1000} = S_0 \exp(-bD),$$

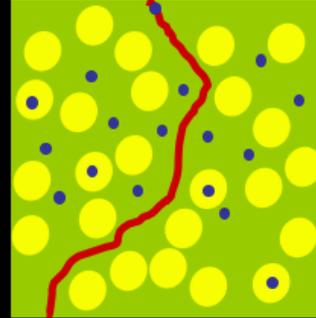
so we can calculate D for each voxel

$$D = [\ln(S_{1000}/S_0)]/-b$$

Tumour Microstructure: Diffusion

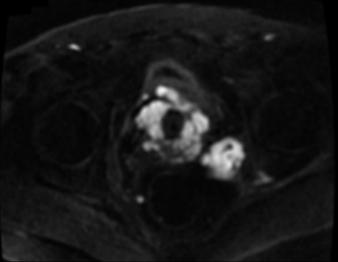
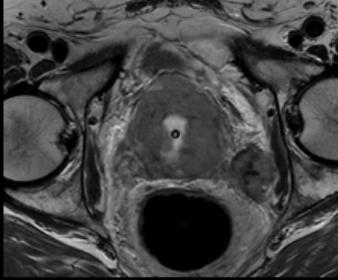


Low cellular density,
defective membranes
e.g. cystic/necrotic
tissue, free diffusion
↓ DWI ↑ ADC

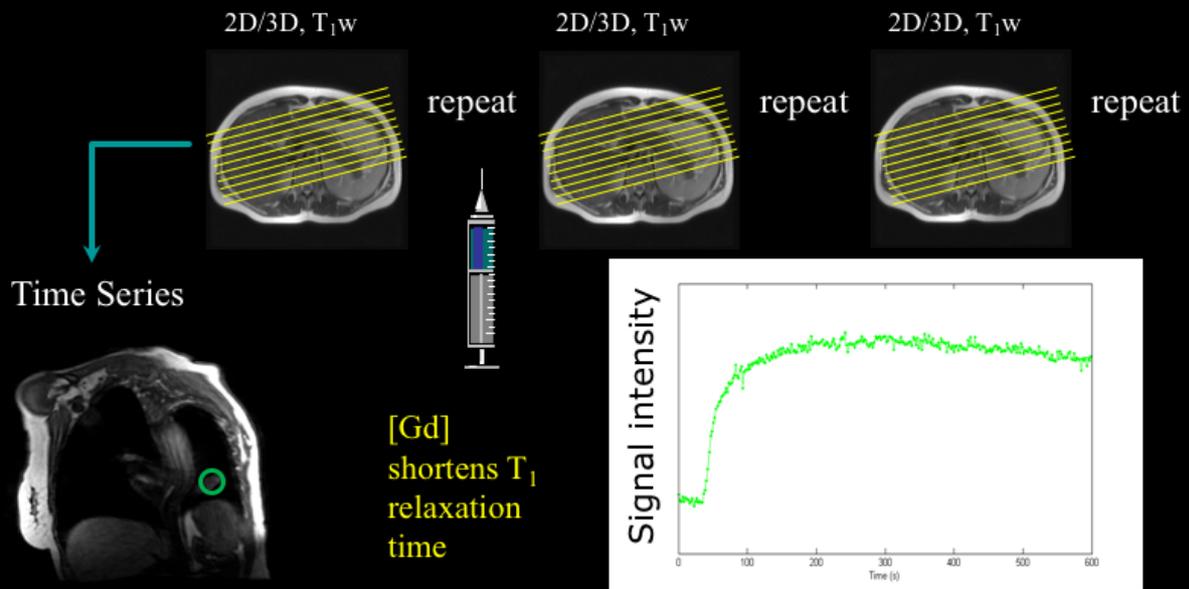


High cellular density,
e.g. solid tumours,
restricted diffusion
↑ DWI ↓ ADC

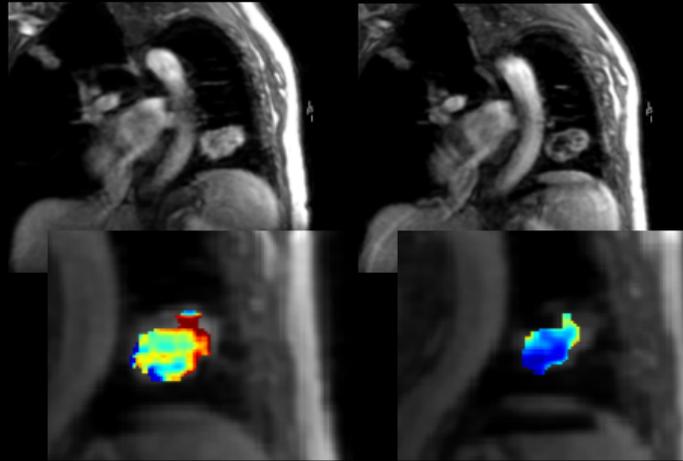
Diffusion Weighted Imaging



Dynamic Contrast enhanced (DCE) MRI



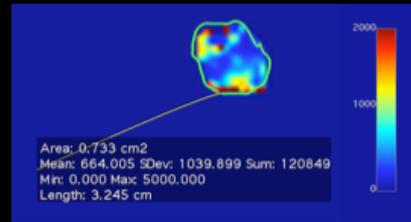
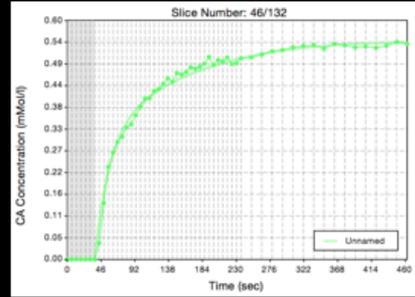
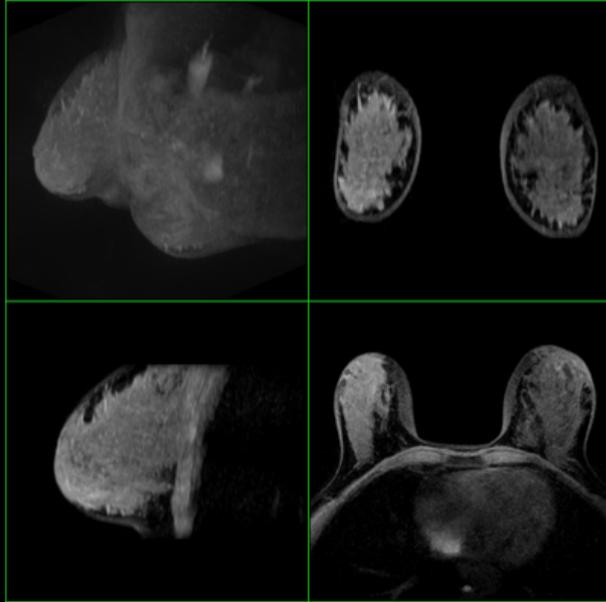
Quantitative DCE-MRI



$K^{trans} = 0.42 \text{ min}^{-1}$

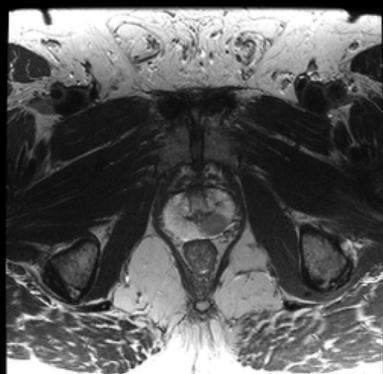
$K^{trans} = 0.34 \text{ min}^{-1}$

Breast DCE-MRI

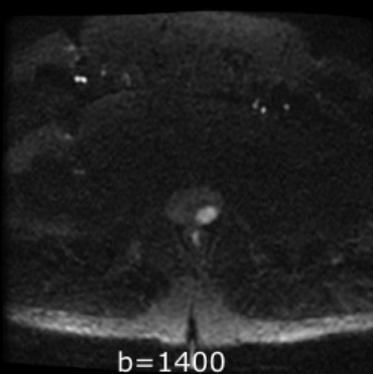


$$K^{\text{trans}} = 0.664 \text{min}^{-1}$$

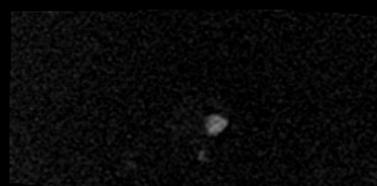
Multicontrast Prostate



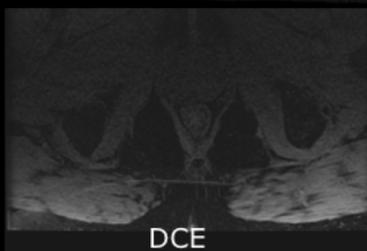
T2w



b=1400



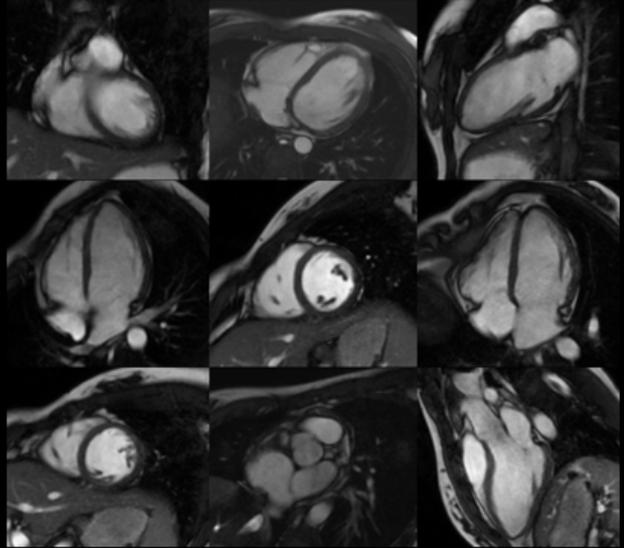
b=2000



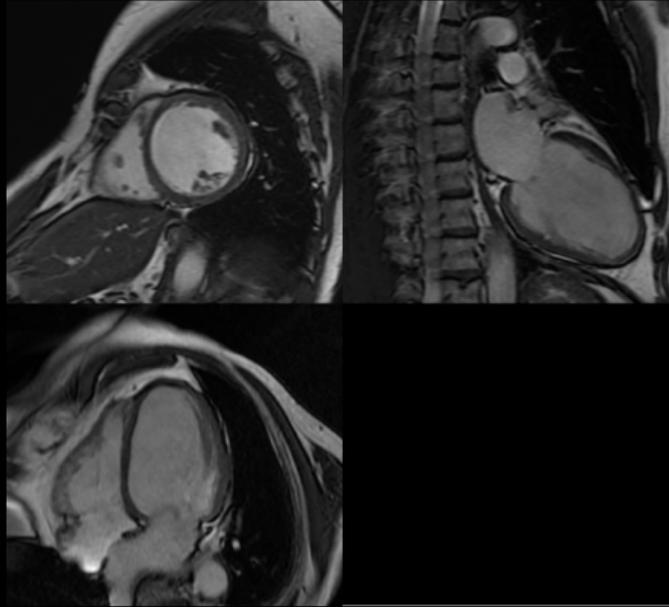
DCE

Functional Cardiac Imaging

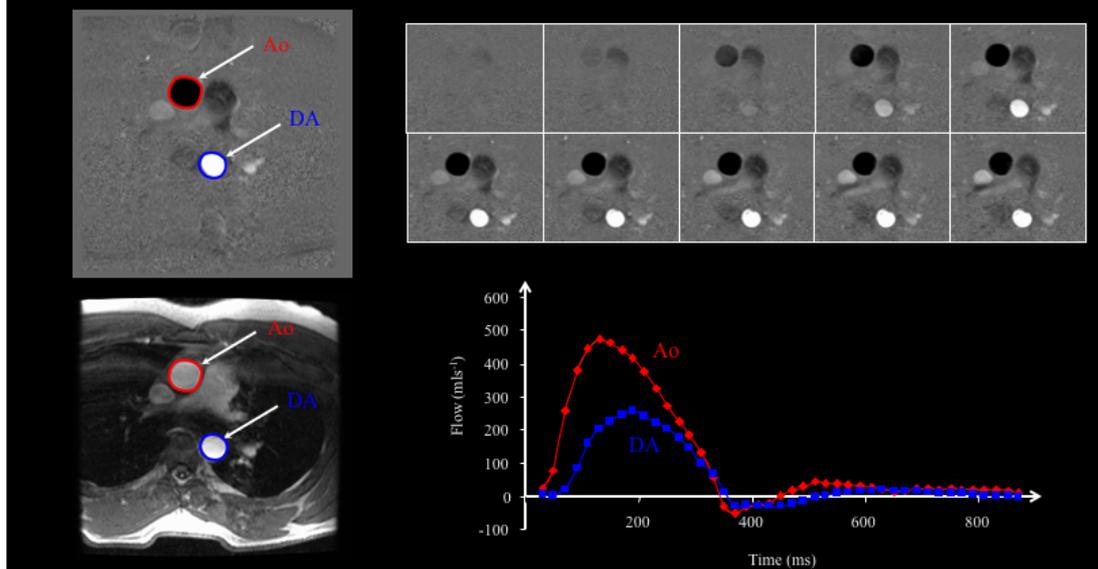
- ▶ “Bright” blood
- ▶ Good blood/myocardial contrast
- ▶ ECG triggered
- ▶ Multi-phase “cine” acquisition
- ▶ Minimal artefacts – breath-hold acquisition
- ▶ Full cardiac cycle coverage



Myocardial Infarction



Phase Contrast Velocity Mapping

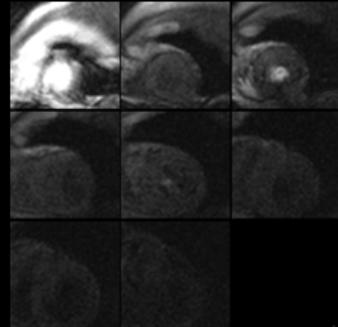
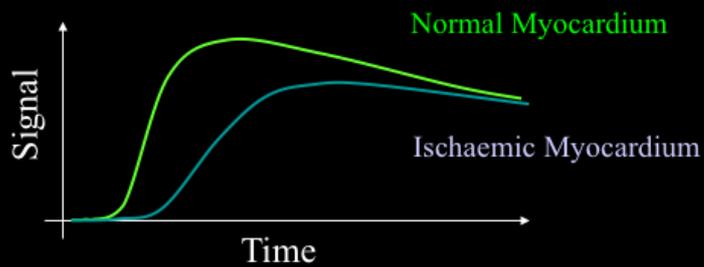


Cine phase contrast imaging through the ascending aorta (Ao). (a) shows the phase/velocity image and (b) shows the magnitude image from a single temporal phase. The velocity in the Ao is shown as shades of mid-grey to black, whilst velocity in the opposite direction in the descending aorta (DA) is shown as shades of mid-grey to white. Regions of interest (ROI) for both vessels have been outlined. (c) shows the first ten temporal phases of the phase/velocity images. (d) shows the flow time curve. The flow at each temporal phase is obtained by multiplying the average velocity in each ROI (mm/s) by the area of the ROI (mm²) to give a flow in mm³ s⁻¹ or ml s⁻¹. Note that only the magnitude of the flow in the DA is shown in (d).

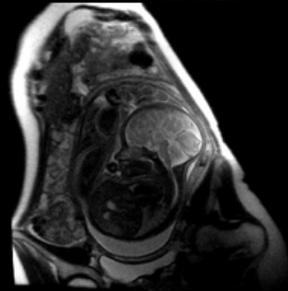
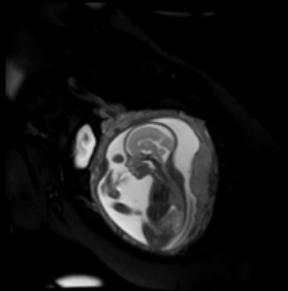
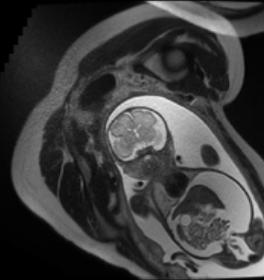
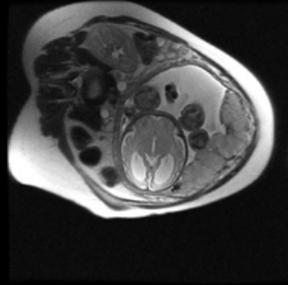
Myocardial Perfusion Imaging

Dynamic contrast-enhanced technique:

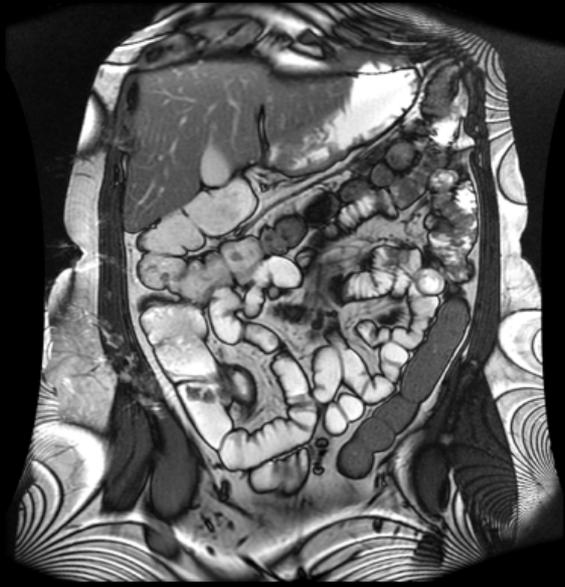
- A bolus of contrast agent (Gd-DTPA) is injected intravenously
- The circulation carries the contrast to the coronary arteries
- Regions of decreased myocardial perfusion have **delayed** and **diminished** enhancement



Fetal



Gut Motility



Learning Outcomes

- After these lectures you should be able to:
 - Explain how nuclear spin gives rise to magnetic resonance
 - Understand the principles of T_1 , T_2 and T_2^* relaxation
 - Explain the principles of MR image formation
 - Describe the spin echo and gradient echo pulse sequences
 - Outline the basic components of an MRI system
 - Understand the safety issues related to MRI
 - Describe some of the clinical applications of MRI